Carbon Emission Consideration in a Fuzzy Inventory Model under Hybrid Payment Policy and Trade Credit Financing

A.Thangam*

*Department of Mathematics, Pondicherry University – Community College, Lawspet , Pondicherry - 605 008, India. Email: thangampucc@gmail.com

Abstract

Global manufacturers are sensitive on the emissions of carbon dioxide and other green house gases while material handling in logistics, transport, production of inventory items and waste disposal in their industries. Today's business environment is mostly associated with permissible delay payments under the situations like Covid-19 Pandemic. In business, the prices of commodities are imprecise due to fluctuations in production and marketing. The purpose of this study is to model an inventory system with hybrid payment strategy under fuzzy inventory costs and fuzzy carbon emission cost. The supplier is offering their retailers permissible delay for the payment of imperishable inventory items. The retailer's profit function is defuzzified using signed distance method. A solution method is proposed to obtain optimal replenishment policies for maximizing the retailer's profit. Several managerial insights are also obtained with several numerical examples.

Keywords; Inventory; Fuzzy ; Carbon Emission; Trade credit.

1. Introduction

While processing the inventory items in an industry, the release of carbon particles into the atmosphere damages the nature and human health due to Global warming. Due to increase in awareness concerning about environment, social and economic growth, the global reports on climate change suggest governments to keep carbon emissions from their countries to zero by 2100. To control green house gases, advanced manufacturing facility such as life cycle assessment (LCA) technology is required. It is used for assessing products' environmental impacts. Eco-friendly materials are utilized for packaging. This will result additional charges to maintain the carbon emission level, that is, carbon – level capping.

Inventory costs such as ordering cost, holding cost, carbon emission control cost, interest rates are imprecise due to the deficiency in definite form of the information. To handle such ambiguity in the model formulation, a fuzzy set theory (Zadeh [27]) plays a key role. Money flow between nations affects the interest rate. [26] discussed that Tumor segmentation required also the identical automatic initialization as regarding the liver. This phase was applied only in order to liver volume, obtained following automatic delineation of lean meats surface: this latter, used to original dataset quantity, was used as a new mask in order to be able to prevent processing overloads and even avoid errors related to be able to arsenic intoxication surrounding tissues delivering similar gray scale droit. [24] discussed that Live wire with Active Appearance model (AAM) strategy is called Oriented Active Appearance Model (OAAM). The Geodesic Graph-cut calculation creates much better division results than some other completely programmed strategies distinguished in writing in the expressions of exactness and period preparing

Due to lack of financial flow, permissible delay period offered by supplier is a type of promotional tool to attract his retailer. The revenue generated through sales can be deposited in interest generating account by the retailer and he can also earn interest using deposited amount. Hybrid payment Policy (Taleizadeh et al.[25]) consists of

(a) Multiple prepayment as advance payment to prevent cancellation of the purchase orders and manage the products. To purchase a special product, retailer may have advance payment for setting up orders.
(b) Supplier offers their retailer a delayed payment to stimulate the demand. It increases sales and promotes the product.

Prepayment is preferred by the retailer when their buyers are not giving up their orders. Sometimes customer prefer to pay the purchasing cost in advance in several installments and get the cooperative profit from the manufacturer when the interest earned rate is more than bank’s capital rate. (Taleizadeh et al.[25]) [22] discussed that Automatic liver tumor segmentation would bigly influence liver treatment organizing strategy and follow-up assessment, as a result of organization and joining of full picture information. Right now, develop a totally programmed technique for liver tumor division in CT picture.

This paper considers a fuzzy inventory model with imperishable items in which supplier is offering hybrid payment policy to his retailer. The inventory cost parameters are treated as fuzzy numbers due to uncertainty in real life business. The purpose of this study is to consider carbon emission cost in the fuzzy inventory model with trade credit and hybrid payment policy and maxiimizing profit in a retail industry.

2 a) Literature review

To face the uncertainty in inventory parameters, fuzzy set theory is used in the following literature. Pertrovic and Sweeney [1] considered the demand rate, lead time and inventory level into triangular fuzzy numbers, and they determined the optimal order quantity with the fuzzy propositions method. Yao et al. [2] developed the Economic Lot Size Production model with customer demand as a fuzzy variable. Yao et al. [3] established a fuzzy inventory system without the backorders in which both the order quantity and the total demand were fuzzified as the triangular fuzzy numbers. Chang [4] created the Economic Order Quantity (EOQ) model with imperfect quality items by applying the fuzzy sets theory, and proposed the model with both a fuzzy defective rate and a fuzzy annual demand. Chang et al. [5] considered the mixture inventory model involving variable lead time with backorders and lost sales. They fuzzified the random lead-time demand to be a fuzzy random variable and the total demand to be the triangular fuzzy number. Based on the centroid method of defuzzification, they derived an estimate of the total cost in the fuzzy sense. Chen *et al*. [6] introduced a fuzzy economic production quantity model with defective products in which they considered a fuzzy opportunity cost, trapezoidal fuzzy cost and quantities in the context of the traditional production inventory model. Maiti [7] developed a multi-item inventory model with stock-dependent demand and two-storage facilities in a fuzzy environment (where purchase cost, investment amount and storehouse capacity are imprecise) under inflation and incorporating the time value of money. Other related articles on this topic can be found in work by Chen and Wang [8], Vujosevic *et al*. [9], Gen et al. [10], Roy and Maiti [11], Ishii and Konno [12], Lee and Yao [13], Yao and Lee [14], Chang et al. [15,16,17], Ouyang et al.[18,19], Yao et al. and other literatures [21,23,25] . [20] discussed about the combination of Graph cut liver segmentation and Fuzzy with MPSO tumor segmentation algorithms. The system determines the elapsed time for the segmentation process.

The rest of this paper is organized as follows. In section 3, notations and assumptions are presented. Section 4 develops a fuzzy inventory model under trade credit financing and carbon emission control. Optimal solution procedures are obtained in section 4. Section 5 presents several numerical examples and sensitivity analysis is also demonstrated graphically. Finally section 6 contains a conclusion of the paper and future research directions.

2 b) Preliminaries

Here, we use some definitions which are well known in fuzzy set theory.

Definition 1 . A **fuzzy set** F on the given universal set X is set of ordered pairs

$$F = \{(x, \mu_F(x)), x \in X\}$$

$where,\quad \mu_F : X \to [0,1]$ is the membership function.

Definition 2. A fuzzy number F is a fuzzy set which satisfies the following conditions:

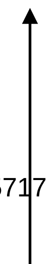
1. F is normal, that is, **there exists** $x \in R$ such that $\mu_F(x) = 1$

2. $\mu_F(x)$ is piece-wise continuous function.

3. F is convex fuzzy set.

Definition 3. A trapezoidal fuzzy number F =(a,d,c,d) is represented with the following membership function

$$\mu_F(x) = \begin{cases} L(x) = \dfrac{x-a}{b-a} & \text{if} \quad a \le x \le b \\ 1 & \text{if} \quad b \le x \le c \\ R(x) = \dfrac{d-x}{d-c} & \text{if} \quad c \le x \le d \\ 0 \ \text{otherwise} & \end{cases}$$



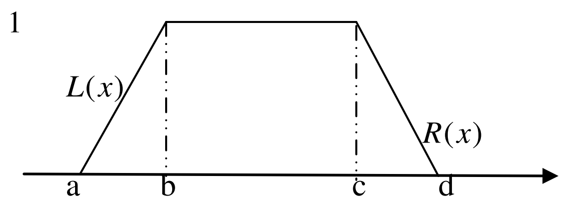


Fig1 : Trapezoidal fuzzy membership function

Arithmetic Operations in fuzzy numbers:

Suppose $A=(a_1,a_2,a_3,a_4)$ and $B=(b_1,b_2,b_3,b_4)$ are two trapezoidal fuzzy numbers, then

$$A\oplus B=(a_1+b_1,a_2+b_2,a_3+b_3,a_4+b_4)$$
$$A\otimes B=(a_1b_1,a_2b_2,a_3b_3,a_4b_4)$$
$$A-B=(a_1-b_4,a_2-b_3,a_3-b_2,a_4-b_1)$$
$$kA=\begin{cases}(ka_1,ka_2,ka_3,ka_4) & \text{if } k>0\\ (ka_4,ka_3,ka_2,ka_1) & if \;\; k\le 0\end{cases}$$
$$\frac{A}{B}=\left(\frac{a_1}{b_4},\frac{a_2}{b_3},\frac{a_3}{b_2},\frac{a_4}{b_1}\right)$$

Signed Distance Method (Yao and Wu [29])

The signed distance method is used to defuzzify the total fuzzy inventory cost. Consider the trapezoidal fuzzy number $F=(a,b,c,d)$. The defuzzified value of F can be calculated as follows:

$$Z(F)=\frac{\text{a+b+c+d}}{4}$$

3.Notations and assumptions

I(t) – Inventory level at time t,

T – Cycle time, (decision variable),

Q - Ordering quantity,

C_r - Carbon emission control cost,

A – Fuzzy Ordering cost,

A_c - Carbon emission quantity in Ordering and Setting up for inventory items,

$\tilde{c}$ – Fuzzy purchasing cost,

g - Fuzzy social cost from vehicle carbon emission,

$\tilde{a}_t$ - Fuzzy transport cost,

d - Distance travelled due to transport of inventory items,

v -- Average velocity,

$\tilde{s}$ - Fuzzy selling price,

$\tilde{h}$ – Fuzzy holding charges per $ per unit item,

h_c - Carbon emission quantity from inventory holding,

ω_0 - Fuzzy cost for disposing the waste,

ω - Fuzzy Waste disposal cost per lot,

θ - Fuzzy rate of waste produced per lot

$\tilde{I}_e$ – Fuzzy Interest rate earned rate per $ per unit item,

$\tilde{I}_p$ - Fuzzy Interest payable rate per $ per unit item

λ - Demand rate per annum

t_M - Retailer's credit period offered by the supplier.

L - Period for advance payments at multiple times

γ - Percentage of purchasing cost prepaid as advance payment

n - Number of equally spaced pre-payments,

Π - Life cycle assessment cost

TP(T) - Retailer's Total Profit as a function of T.

Assumptions:

1. Inventory ordering policy obeys Carbon emission control.
2. Items are non-deteriorating with respect to time.
3. Replenishment of items in the inventory is instantaneous.

4. The purchasing cost, carbon emission control cost selling price, holding cost, interest earned rate and interest payable rates are considered as trapezoidal fuzzy numbers.
5. No shortage is allowed.
6. The Supplier is offering hybrid payment strategy to his retailer.
7. Supplier provides the retailer a fixed permissible delay period t_M to settle the accounts. If the retailer does not pay at time t_M, he has to pay penalty at rate I_p.
8. The prepayments by the retailer are payable at multiple equal sizes.
9. Demand rates are assumed to be constant.

4. (A) CRISP MODEL

Here, the inventory system is described by deterministic inventory parameters and total profit function for the retailer is derived mathematically.

The annual total profit of the retailer's inventory system under carbon emission control is

TP (T) = Annual Sales Revenue + Annual Interest earned -

[Annual Ordering cost including carbon emission control+ Annual Purchase cost+ Annual Holding cost including carbon emission control+ Annual Interest Payable+ Social cost+ Transport cost+ Waste disposal cost] ------ (1)

The inventory level is described by the following equations

$$I(t)=\begin{cases} Q & \text{if } t_x=0 \\ Q-\lambda t_x & \text{if } 0<t_x<T \\ 0 & \text{if } t_x=T \end{cases} \qquad \text{---- (2)}$$

where $Q=\lambda T$.

a) Annual sales revenue = $s\lambda$

b) Annual Ordering cost including carbon emission control = $\left(A+C_rA_c\right)/T$

c) Annual Purchase cost = $c\lambda$

d) Annual holding cost including carbon emission control $= \frac{(h + C_r H_r)}{T} \int_o^T I(t)\,dt$

e) Annual Life cycle assessment cost for reusing raw material waste = $\frac{L_c}{T}$

f) Cost of transporting items and Social cost due to carbon emission = $\frac{1}{T}\left[a_t + \frac{gd}{v}\right]$

g) Waste disposal cost = $\frac{\omega_0}{T} + \frac{\omega\theta}{\lambda}$

h) Depending upon the values of credit period t_M, cycle time T and prepayment time γT, the following scenarios are considered to calculate annual interest payable and annual interest earned:-

Case 1 : if $\frac{t_M}{\gamma} \le T$

Here, the retailer is offered to prepay γ percentage of purchasing cost in multiple prepayments and $(1-\gamma)$ percentage of purchase cost in trade credit policy. [28] discussed that Liver tumor division in restorative pictures has been generally considered as of late, of which the Level set models show an uncommon potential with the advantage of overall optima and functional effectiveness. As per the inventory model (Taleizadeh(2020)),

Annual Interest Payable is

$$IP_{1,1} = \gamma c I_p \frac{\lambda T(n+1)L}{2n}$$

Annual Interest earned is

$$IE_{1,1} = (1-\gamma) s I_e \frac{\lambda t_M}{T}$$

Case 2: if $t_M \le T \le \frac{t_M}{\gamma}$

Annual Interest payable due to prepayments $IP_{2,1} = \gamma c I_p \frac{\lambda T(n+1)L}{2n}$

Annual Interest payable due to permissible delay payments $IP_{2,2} = c I_p \frac{\lambda (T - t_M)^2}{2T}$

Annual Interest earned in case 2

$$IE_{2,1} = (1-\gamma)sI_e \frac{\lambda t_M}{T}$$

Case 3: if $t_M \geq T$

Annual Interest payable due to prepayments

$$IP_{3,1} = \gamma c I_p \frac{\lambda T(n+1)L}{2n}$$

Annual Interest earned

$$IE_{3,1} = (1-\gamma)sI_e\lambda$$
$$IE_{3,2} = (1-\gamma)sI_e\lambda(t_M - T)$$

From the above discussions and Eq.(1), the total profit can be estimated as

$$TP(T) = \begin{cases} TP_1(T) & \text{if } T \geq \dfrac{t_M}{\gamma} \\ TP_2(T) & \text{if } t_M \leq T \leq \dfrac{t_M}{\gamma} \\ TP_3(T) & \text{if } 0 \leq T \leq t_M \end{cases} \quad \text{-- -- (3)}$$

where

$$TP_1(T) = \left[(s-c)\lambda - \frac{\omega\theta}{\lambda} - \gamma c I_p \lambda \frac{n+1}{n} L + c I_p \lambda t_M\right] + \frac{1}{2T}\left[2(1-\gamma)sI_e\lambda t_M - \left(2\Pi + 2a_t + \frac{2gd}{v} + 2\omega_0 + 2L_c\right) - 2(A + C_r A_c) + cI_p\lambda t_M{}^2\right] - \left[h + C_r h_r + cI_p\right]\frac{\lambda T}{2}$$

---(3.1)

$$TP_2(T) = \left[(s-c)\lambda - \frac{\omega\theta}{\lambda} - \gamma c I_p \lambda \frac{n+1}{n} L \right] + \frac{1}{\mathrm{T}} \left[(1-\gamma) s I_e \lambda t_M^2 - (A + C_r A_c + \Pi + a_t + \frac{gd}{v} + \omega_0 + L_c) \right]$$
$$- \left[(h + C_r h_c) \frac{\lambda T}{2} \right]$$

----(3.2)

$$TP_3(T) = \left[(s-c)\lambda - \frac{\omega\theta}{\lambda} + sI_e\lambda + (1-\gamma) s I_e \lambda t_M - \gamma c I_p \lambda \frac{n+1}{n} L \right]$$
$$- \frac{\lambda \mathrm{T}}{2} \left[2(1-\gamma) s I_e + (h + C_r h_c) \right] - \frac{1}{T} \left[A + C_r A_c + \Pi + a_t + \frac{gd}{v} + \omega_0 + L_c \right]$$

---- (3.3)

4 (B). Fuzzy model

Here, the crisp inventory parameters are treated as trapezoidal fuzzy numbers.

$\tilde{c} = (c_1, c_2, c_3, c_4)$, $\tilde{s} = (s_1, s_2, s_3, s_4)$, $C_r = (C_{r1}, C_{r2}, C_{r3}, C_{r4})$, $h = (h_1, h_2, h_3, h_4)$, $\theta = (\theta_1, \theta_2, \theta_3, \theta_4)$ $h_c = (h_{c1}, h_{c2}, h_{c3}, h_{c4})$, $I_e = (Ie_1, Ie_2, Ie_3, Ie_4)$ $I_p = (Ip_1, Ip_2, Ip_3, Ip_4)$. $A = (A_1, A_2, A_3, A_4)$, $Ac = (A_{c1}, A_{c2}, A_{c3}, A_{c4})$, $a_t = (a_{t1}, a_{t2}, a_{t3}, a_{t4})$, $\omega_0 = (\omega_{01}, \omega_{02}, \omega_{03}, \omega_{04})$, $\omega = (\omega_1, \omega_2, \omega_3, \omega_4)$, $\Pi = (\Pi_1, \Pi_2, \Pi_3, \Pi_4)$

The total profit is

$$TP(T) = \begin{cases} TP_1(T) & \text{if } \mathrm{T} \ge \frac{\mathrm{t_M}}{\gamma} \\ \mathrm{TP_2(T)} & \text{if } \mathrm{t_M} \le \mathrm{T} \le \frac{\mathrm{t_M}}{\gamma} \\ TP_3(T) & \text{if } 0 \le T \le t_M \end{cases}$$

-- (4)

where

$$TP_1(T)=\left[\lambda\left(\tilde{s}\Theta\tilde{c}\right)-\frac{\omega\otimes\theta}{\lambda}-\gamma\lambda L\frac{n+1}{n}\tilde{c}\otimes I_p+\lambda t_M\tilde{c}\otimes I_p\right]$$
$$+\frac{1}{2\mathrm{T}}\left[2(1-\gamma)\lambda t_M\tilde{s}\otimes I_e-\left(2\Pi+2a_t+\frac{2d\,g}{v}+2\omega_0+2L_c\right)-2(A+C_r\otimes A_c)+\tilde{c}\otimes I_p\lambda t_M{}^2\right]$$
$$-\frac{\lambda T}{2}\left[h+C_r\otimes h_r+\tilde{c}\otimes I_p\right]$$

---(4.1)

$$TP_2(T)=\left[\lambda\left(\tilde{s}\Theta\tilde{c}\right)-\frac{\omega\theta}{\lambda}-\gamma\lambda L\frac{n+1}{n}\tilde{c}\otimes I_p\right]+\frac{1}{\mathrm{T}}\left[\begin{array}{l}(1-\gamma)\lambda t_M^2\tilde{s}\otimes I_e-(A+C_r\otimes A_c+\Pi+a_t\\+\frac{d}{v}g+\omega_0+L_c)\end{array}\right]$$
$$-\frac{\lambda T}{2}\left[(h+C_r h_c)\right]$$

--- (4.2)

$$TP_3(T)=\left[\left(\tilde{s}\Theta\tilde{c}\right)\lambda-\frac{\omega\otimes\theta}{\lambda}+\tilde{s}\otimes I_e\lambda+(1-\gamma)\lambda t_M\tilde{s}\otimes I_e\ -\gamma\lambda L\frac{n+1}{n}\tilde{c}\otimes I_p\right]$$
$$-\ \frac{\lambda\mathrm{T}}{2}\left[2(1-\gamma)\tilde{s}\otimes I_e+(h+C_r\otimes h_c)\right]-\frac{1}{T}\left[A+C_r\otimes A_c+\Pi+a_t+\frac{d}{v}g+\omega_0+L_c\right]$$

---(4.3)

Applying signed distance method, the defuzzified value of total profit is

$$F[TP(T)]=\begin{cases}\mathrm{F}[TP_1(T)\quad \text{if } \mathrm{T}\geq\dfrac{\mathrm{t_M}}{\gamma}\\ \mathrm{F}[\mathrm{TP}_2(\mathrm{T})]\ \ \text{if}\ \ \mathrm{t_M}\leq\mathrm{T}\leq\dfrac{\mathrm{t_M}}{\gamma}\\ F[TP_3(T)]\quad \text{if } 0\leq T\leq t_M\end{cases}$$

where

$$F[TP_1(T)]=\frac{\lambda T}{8}\left[\sum_{i=1}^{4}(h_{5-i}+C_{r5-i}h_{r5-i}+C_{5-i}I_{p5-i})\right]+\frac{1}{4T}\left[(1-\gamma)\lambda t_m\sum_{i=1}^{4}s_iI_{ei}+\sum_{i=1}^{4}\left(\begin{array}{l}\Pi_{5-i}+a_{t5-i}+\frac{d}{v}g_{5-i}+\omega_{05-i}+L_{c5-i}\\+A_{5-i}+A_{c5-i}C_{r5-i}+\frac{\lambda t_m{}^2}{2}c_iI_{pi}\end{array}\right)\right]$$
$$+\frac{1}{4}\left[\lambda\sum_{i=1}^{4}(s_i-c_{5-i})+\lambda t_m\sum_{i=1}^{4}c_iI_{pi}+\frac{\lambda\gamma(n+1)L}{n}\sum_{i}c_{5-i}I_{p5-i}+\frac{1}{\lambda}\sum_{i}\omega_{5-i}\theta_{5-i}\right]$$

$$F[TP_2(T)]=\frac{\lambda T}{8}\left[\sum_{i=1}^{4}(h_{5-i}+C_{r5-i}h_{r5-i})\right]+\frac{1}{4T}\left[(1-\gamma)\lambda t_m{}^2\sum_{i=1}^{4}s_iI_{ei}+\sum_{i=1}^{4}\left(\begin{array}{l}\Pi_{5-i}+a_{t5-i}+\frac{d}{v}g_{5-i}+\omega_{05-i}+L_{c5-i}\\+A_{5-i}+A_{c5-i}C_{r5-i}\end{array}\right)\right]$$
$$+\frac{1}{4}\left[\lambda\sum_{i=1}^{4}(s_i-c_{5-i})+\frac{\lambda\gamma(n+1)L}{n}\sum_{i=1}^{4}c_iI_{pi}+\frac{1}{\lambda}\sum_{i}\omega_{5-i}\theta_{5-i}\right]$$

$$F[TP_3(T)]=\frac{\lambda T}{8}\left[\sum_{i=1}^{4}(h_{5-i}+C_{r5-i}h_{r5-i}+2(1-\gamma)(s_{5-i}I_{e5-i})\right]+\frac{1}{4T}\left[\sum_{i=1}^{4}\left(\begin{array}{l}\Pi_{5-i}+a_{t5-i}+\frac{d}{v}g_{5-i}+\omega_{05-i}+L_{c5-i}+\\A_{5-i}+A_{c5-i}C_{r5-i}\end{array}\right)\right]$$
$$+\frac{1}{4}\left[\lambda\sum_{i=1}^{4}(s_i-c_{5-i}+s_iI_{ei}+(1-\gamma)\lambda t_M{}^2s_iI_{ei})+\frac{1}{\lambda}\sum_{i=1}^{4}\omega_{5-i}\theta_{5-i}+\frac{\lambda\gamma(n+1)L}{n}\sum_{i=1}^{4}c_{5-i}I_{p5-i}\right]$$

Let

$$\Phi_{11}=\sum_{i=1}^{4}(h_{5-i}+C_{r5-i}h_{r5-i}+C_{5-i}I_{p5-i})$$

$$\Phi_{12}=(1-\gamma)\lambda t_m\sum_{i=1}^{4}s_iI_{ei}+\sum_{i=1}^{4}\left(\begin{array}{l}\Pi_{5-i}+a_{t5-i}+\frac{d}{v}g_{5-i}+\omega_{05-i}+L_{c5-i}+\\A_{5-i}+A_{c5-i}C_{r5-i}+\frac{\lambda t_m{}^2}{2}c_iI_{pi}\end{array}\right)$$

$$\Phi_{13}=\lambda\sum_{i=1}^{4}(s_i-c_{5-i})+\lambda t_m\sum_{i=1}^{4}c_iI_{pi}+\frac{\lambda\gamma(n+1)L}{n}\sum_{i}c_{5-i}I_{p5-i}+\frac{1}{\lambda}\sum_{i}\omega_{5-i}\theta_{5-i}$$

$$\Phi_{21} = \sum_{i=1}^{4} (h_{5-i} + C_{r5-i} h_{r5-i})$$

$$\Phi_{22} = (1-\gamma)\lambda t_m{}^2 \sum_{i=1}^{4} s_i I_{ei} + \sum_{i=1}^{4} \begin{pmatrix} \Pi_{5-i} + a_{t5-i} + \frac{d}{v} g_{5-i} + \omega_{05-i} + L_{c5-i} + \\ A_{5-i} + A_{c5-i} C_{r5-i} \end{pmatrix}$$

$$\Phi_{23} = \lambda \sum_{i=1}^{4} (s_i - c_{5-i}) + \frac{\lambda\gamma(n+1)L}{n} \sum_{i=1}^{4} c_i I_{pi} + \frac{1}{\lambda} \sum_{i} \omega_{5-i} \theta_{5-i}$$

$$\Phi_{31} = \sum_{i=1}^{4} (h_{5-i} + C_{r5-i} h_{r5-i} + 2(1-\gamma)(s_{5-i} I_{e5-i})$$

$$\Phi_{32} = \sum_{i=1}^{4} \begin{pmatrix} \Pi_{5-i} + a_{t5-i} + \frac{d}{v} g_{5-i} + \omega_{05-i} + L_{c5-i} + \\ A_{5-i} + A_{c5-i} C_{r5-i} \end{pmatrix}$$

$$\Phi_{33} = \left[\lambda \sum_{i=1}^{4} (s_i - c_{5-i} + s_i I_{ei} + (1-\gamma)\lambda t_m{}^2 s_i I_{ei}) + \frac{1}{\lambda} \sum_{i=1}^{4} \omega_{5-i} \theta_{5-i} + \frac{\lambda\gamma(n+1)L}{n} \sum_{i=1}^{4} c_{5-i} I_{p5-i} \right]$$

The defuzzified value of total profit is

$$F[TP(T)] = \begin{cases} \mathrm{F}[\, TP_1(T)] & \text{if } \mathrm{T} \geq \frac{\mathrm{t_M}}{\gamma} \\ \mathrm{F}[\, \mathrm{TP_2(T)}] & \text{if } \mathrm{t_M} \leq \mathrm{T} \leq \frac{\mathrm{t_M}}{\gamma} \\ F[TP_3(T)] & \text{if } 0 \leq T \leq t_M \end{cases}$$

where

$$F[TP_1(T)] = \frac{\lambda T}{8} \Phi_{11} + \frac{1}{4T} \Phi_{12} + \frac{1}{4} \Phi_{13}$$

$$F[TP_2(T)] = \frac{\lambda T}{8} \Phi_{21} + \frac{1}{4T} \Phi_{22} + \frac{1}{4} \Phi_{23} \quad ,$$

$$F[TP_3(T)] = \frac{\lambda T}{8} \Phi_{31} + \frac{1}{4T} \Phi_{32} + \frac{1}{4} \Phi_{33}$$

To find the optimum value of cycle time T, the optimality conditions are:

$$\frac{d}{dT}\left(F\left[TP_1\right]\right)=0, \frac{d}{dT}\left(F\left[TP_2\right]\right)=0, and\ \frac{d}{dT}\left(F\left[TP_3\right]\right)=0 \quad .$$

From these conditions,

$$optimal\ \text{ cycle time } T^* = \begin{cases} T_1^* & \text{if } T \geq \dfrac{t_M}{\gamma} \\ T_2^* & \text{if } t_M \leq T \leq \dfrac{t_M}{\gamma} \\ T_3^* & \text{if } 0 \leq T \leq t_M \end{cases}$$

where

$$T_1^* = \sqrt{\frac{2\Phi_{12}}{\lambda\Phi_{11}}} \quad ,$$

$$T_2^* = \sqrt{\frac{2\Phi_{22}}{\lambda\Phi_{21}}} \quad ,$$

$$T_3^* = \sqrt{\frac{2\Phi_{32}}{\lambda\Phi_{31}}}$$

5. Numerical Example

To illustrate the proposed fuzzy inventory model, the fuzzy inventory cost parameter values are considered in dollar as below:

$\tilde{c} = (10,12,14,16)$, $\tilde{s} = (18,20,22,24)$, $C_r = (0 . 2 , 0 . ,$ $h = (3,3.5,4,4.5)$,

$\theta = (0.1,0.15,0.2,0.25)$, $h_c = (0.5,1,1.5,2)$, $I_e = (0.05,0.1,0.15,0.2)$

$I_p = (0.02,0.04,0.06,0.08)$ $A = (100,120,140,160)$, $A_c = (60,80,100,120)$, $a_t = (5,7,9,11)$,

$\omega_0 = (0.8,1,1.2,1.4)$, $\omega = (0.5,1,1.5,2)$, $\prod = (0.5,1,1.5,2)$

$t_M = 0.1$ and $\lambda = 300$. The optimal solutions are obtained as below:

Optimal cycle time	=	0.6643 unit time,
Optimal order Quantity	=	199 items
Maximum Profit	=	$ 11732

6. Conclusions

In this paper, a fuzzy inventory model with imperishable items is developed under carbon emission cost consideration. The retailer is allowed to avail multiple prepayments as well as trade credit policy. Profit function for inventory system is derived mathematically and optimization procedures are also obtained. Several inventory related parameters are fuzzified by assuming triangular fuzzy numbers. Using fuzzy arithmetic operations and signed distance method, optimal decision policy for the retailer is explored. Finally numerical example is presented to illustrate mathematical solutions

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