MATHEMATICAL THEORY AND NUMERICAL CONCEPTS OF FLUID FLOWS

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Abstract - Fluid flow is an important criterion in all fields of science and engineering. The mathematical theory of fluid flow is discussed in this paper. The numerical concept of various types of flow is analyzed. This paper gives a broad view of the fluid flows mathematically and numerically. The literature reviews of fluid flow are discussed.

Keywords - Fluid Flow, Mathematical theory, Numerical concepts.

I. INTRODUCTION

Fluid flow is a part of the fluid mechanics and deals with fluid dynamics. Fluid such as gases and liquids in motion is called fluid flow. The fluid in motion is subjected to unbalanced forces. This motion will be continued as long as unbalanced forces are applied. It is also defined for the molecules which are moving freely, as a state of matter which does not have a constant relationship with the other molecules in space. Some common fluids are Liquid: Blood, IV,.. Gases: Oxygen, Nitrous oxide, etc., Vapour: Transition from liquid to gas, sublimate: Transition from solid to gas by passing liquid state like dry ice and iodine.

Flow is defined as the quantity of fluid (gas/liquid/ vapour state or sublimate) that passes a point per unit time. A simple equation to represent the flow F=Quantity (Q)/ Time (t)

Flow is also written as ΔQ (rate of change of quantity)

II. FLUID FLOWS

The fluid flows are categorized as below. Some of the characteristics reflect properties the liquid Itself and other focus on how the fluid is moving.

Steady or Unsteady flow: In steady flow the fluid characteristics at a point like velocity, pressure density, etc., does not change with respect to time. In the unsteady flow the fluid properties change with respect to time at a point.

Compressible or Incompressible flow: It is a branch of fluid mechanics and continuum mechanics that deals with flow having significant changes in fluid density of gases and liquid. To differentiate between compressible and incompressible flow in air Mach Number which is defined as the ratio of the speed of the flow to the speed of the sound., should always be greater than about 0.3 since the density change is greater than 5% before significant compressibility of flow is related to high speed aircraft, rocket motors, jet engines, high speed entry into a planetary atmosphere, gas pipelines commercial applications like abrasive blasting and in all other areas.

In the study of Compressible flow, the density of the fluid changes and it is not constant at any point. In the Incompressible flow, the density of fluid changes from point to point where density is constant.. This flow refers to which the material density is constant within the fluid with a small change in the volume that moves with the flow velocity. Incompressibility is also said as the divergence of the flow velocity is zero.

The change in pressure and temperature is sufficiently small such that changes in density is negligible for which it can be modeled as an incompressible flow else it is compressible flow. Mathematical expression implies the density of the fluid parcel does not change as it moves in the fluid flow field which is given by $\frac{D\rho}{Dt} = 0$ where $\frac{D}{Dt}$ is called as material derivative which is the addition of local and convective derivatives and when the fluid has uniform density, the constraints is simplified using the governing equations. Based on the Mach number of the fluid flow we determine the compressibility or incompressibility of the fluid.

Newtonian and Non – Newtonian fluids: The fluids exert resistance to deformation of the nearby fluid parcels moving at varying velocities exert viscous forces on each other.

Strain rate is defined as the velocity gradient which has dimensions. The stress due to the viscous forces is linearly related to the strain rate are such fluids are defined as Newtonian fluids. The coefficient is called the viscosity of the fluids and this property is independent of the strain rate.

Non-Newtonian fluids are non-linear stress-strain behavior.

Uniform and Non Uniform flow: In uniform flow the velocity at given time does not change with respect to space which is considered as the length of direction of the flow.

In non uniform flow, the velocity at any time consequent changes with respect to space.

Viscous or non viscous flow: Inviscid flow is called as ideal fluid which is assumed to have no viscosity. In fluid dynamics the problem are solved with simplified assumption of an in-viscid flow. The flow of fluids with low values of viscosity agree with the in-viscid flow which is close to the fluid boundary where boundary layer plays a significant role Newton's second law describes the dynamic of fluid parcels and subjected to inertial effects The magnitude of the inertial effects compared to the magnitude of viscous effects by the quantity of Reynolds number. If $Re \ll 1$ indicates the viscous forces are very strong when compare to inertial forces where in that case the inertial force is neglected and the flow regime is called Stokes or creeping flow. If $Re \gg 1$, inertial effects have more effect on the velocity field than the viscous effects. Navier Stoke Equations gets simplified to Euler's equation in the elimination of viscosity. Bernoulli's equation is obtained by integrating the Euler's equation along streamline in an in-viscid flow and ir-rotational flow where in it is called as potential flows since the velocity field is expressed as the gradient of a potential energy expression.

Viscosity is not neglected near solid boundaries since no-slip condition generates a thin region of large strain rate, and the boundary layer in which viscosity effects dominate and vorticity is generated. The net force on the bodies is calculated by viscous flow equations and the limitation is called as D'Alembert's paradox.

Isothermal flow: It is a model of compressible fluid flow where the flow remains at same temperature while flowing in the conduit. Heat is transferred through the walls of the conduit is affected by frictional heating into the flow. Since there is a change in the velocity, we have a change in the stagnation temperature. The flow is choked at and not at Mach number equal to one as in the case of fanno flow. It is applied to real gases as well as ideal gases.

III. SPECIAL TYPES OF FLUID FLOWS

The special types of fluid flows are discussed below.

Couette flow: It is a laminar flow of a viscous fluid between the two parallel plates in a space and one of which is moving relative to the other. The flow is possible by virtue of viscous drag force acting on the fluid and the applied pressure gradient parallel to the plates. This type of flow is named after a professor of physics Maurice Marie Alfred Couette, French university of Angers in the late 19^{th} century.

Rotational flow: The fluid particles along the stream lines rotate about their own axis.

Irrotational flow: The fluid particles along the stream lines do not rotate about their own axis.

Cavitation flow: It is the formation of vapour cavities in the liquid. Free zones are created due to the consequence of forces acting upon the liquid. It usually occurs when a liquid is subjected to rapid changes in the pressure that caused the cavity formation where pressure is relatively low. It generates an intense shock wave when subjected to high pressure.

Free Molecular flow: In fluid dynamics, the free molecular flow is where the mean free path of the molecules is larger than the size of the chamber or the object under test. In this flow, the pressure of the remaining gas can be considered as zero. For residual pressure, boiling point is not a dependent variable. It can be considered as the individual particles moving in the straight lines. Speaking practically, vapour state cannot move with the bends to move into the space behind obstacles they hit the tube wall.

Aerodynamic force: It is exerted on the body by the axis or some other gas where the body is immersed and it is the relative motion between the body and the gas.
It is because of the two reasons. The normal force is due to the pressure on the surface of the body, and the shear force is due to the viscosity of the gas also known as friction.

Laminar flow: The fluid particle moves along the well defined path of stream line. It is also called stream flow occurs when a fluid flow in parallel layers with no disruption between the layers. The fluid tends to flow at low velocities without lateral mixing and adjacent layers. In laminar flow the motion of the particles is in orderly fashion and all the particles flow in a straight line and parallel to the pipe walls. It is characterized by high momentum diffusion and low momentum convection,

Turbulent flow: The flow in which the fluid undergoes irregular fluctuations or mixing in contrast to the laminar flow. The speed of the fluid at a point is continuously undergoing changes both in magnitude and direction. The turbulence is characterized with the following features like Irregularity, Diffusivity, Rotationality and Dissipation, The fluid flow moves in very irregular paths or zig-zig way with the fluctuating velocity at a point. The characteristic which enhances the mixing and increasing the rate of mass, momentum, and energy transports is called as diffusivity and it described by diffusion coefficient which is defined as relation between a turbulent flux and the gradient of a mean variable that is for molecular transport. It is an approach for the quantitative analysis of turbulent flows and many models have been formed. Random walk principle with Richardon's four-third power law and variation in the Elder's formula is governed to measure coefficient for larger bodies

Due to the conservation of angular momentum, turbulent flow is characterized by three dimensional vortex generation called as vortex stretching. It is mechanism on which the turbulence energy cascade establishes the structure and it thins the vortices in the direction perpendicular to the direction of stretching because of conservation of volume by the element of the fluid. Turbulence causes formation of eddies in different length scales. The scale at which it happens is called Kolmogorov length scale

. The onset of turbulence can be analysed by the Reynolds number defined as the ratio of the force of inertia to viscous forces in the fluid subjected to relative internal movement due to the boundary layer. Reynolds number is used to determine dynamic similitude between the fluid flow. Regimes of laminar and turbulent flow are given byLaminar flow is characterized with smooth constant fluid motion at low Reynolds number and viscous forces are dominant Turbulent flow produce eddies, vortices, flow instabilities with high Reynolds number and with the inertial forces.

Subsonic, Supersonic and hypersonic flow: In turbo machines the flow occurs at high fraction of Mach number with M=1 called as transonic flows or in the excess of it is called supersonic or hypersonic flows. At these points of in- stabilities, shockwaves and non equilibrium chemical behavior due to ionization are because of transonic flow, supersonic flow and hypersonic flow respectively.

Reactive and non-reactive flows: The flow which is chemically reactive and has its application in wide areas like propulsion devices, rockets, jet engines, fire and safety hazards etc., The conservation of individual species has to be derived by solving the simultaneous equations of kinetics

IV. MATHEMATICAL CONCEPTS OF FLUID FLOW

The axioms of fundamental laws are the conservation of mass, conservation of energy, and conservation of linear momentum. The continuum principle of fluid is continuous and not discrete. The properties of fluids like pressure, density, temperature and flow velocity are defined at even small points in the space continuously from one point to another. The momentum equations for Newtonian fluids are the Navier-Stoke Equations which is defined as the non linear set of differential equations that describes fluid flow in which stress depends linearly or flow velocity gradients and pressure. Using Computational Fluid Dynamics the expressions can be made in a simpler form to solve. In the thermodynamic equation of state, the pressure is a function of other thermodynamic variables which describes the entire problem. The perfect gas equation $is\ p = \frac{\rho R_u T}{M}$, where ρ is density, p is pressure, T is absolute temperature, R_u is gas constant and M is Molar mass.. The CFD uses Euler's equation away from the body and the boundary layer equation in a region close to the body, wherein the matching of solutions is done by the method of matched asymptotic expansions.

A. Conservation laws

The formulation of conservation laws helps to describe the change of mass, momentum or energy within the control volume. It applies Stoke theorem to the infinitesimally small volume within the flow.

Conservation of Mass: The rate of change of fluid mass inside a control volume is equal to the net rate of fluid flow into the volume and the integral form of

Continuity equation is given by $\frac{\partial}{\partial t}\iiint \rho dv = \iint \rho u.ds$ where u is the flow velocity vector and ρ is the density of the fluid. By divergence theorem the continuity equation is given by

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho u) = 0$$

Conservation of Momentum: The change in the momentum of the fluid within the control volume is because of the net flow of momentum into the volume and the external forces acting on the fluid within the volume

$$.\frac{\partial}{\partial t}\iiint \rho v dv = -\iint(\rho u.ds\,)n - \iint p\, f_{body}\, ds + \iiint \rho\, dv + f_{surfaced}$$

The volume is reduced to a infinitesimal small point and the surface force and body force are accounted as the total force F which can be an expression of frictional or gravitational force acting at a point in the fluid flow. $\frac{\mathrm{D}u}{\mathrm{D}t} = F - \frac{\nabla p}{\rho}$, which is a vector equation in three dimensional flow can be expressed as a scalar equation in three coordinate directions. For the viscous, compressible flows it is represented as a Navier Stoke equation

Conservation of Energy: Energy can be converted or transformed from one to another but the total energy in a closed system is always remains closed. The representation is given by $\rho\frac{Dh}{Dt} = \frac{D\rho}{Dt} + \nabla.(k \uparrow \nabla T) + \emptyset,$ where h is enthalpy. $\emptyset$ is called as viscous dissipation function, T is the temperature and k is thermal conductivity of the fluid. By the second law of thermodynamics the viscosity cannot create energy within the control volume as the dissipation is always positive.

V. NUMERICAL CONCEPTS OF FLUID FLOW

Table 1 explains the various numerical approximations associated with the fluid flow

Table 1

Approximations	Purpose
Boussinesq	It neglects variation in density except to calculate buoyancy forces. It is used in free convection problems where the density changes are very small

Slender Body Theory	It is used in Stoke theorem to estimate the force on .or flow field around an object in viscous fluid.
Lubrication Theory & Hele-shaw flow	If certain terms in the equations are negligible we use large aspect ratio
Shallow-water equations	It is used to describe a layer of relatively inviscid fluid with a free surface where the surface gradient is very small
Darcy Law	It is used for the flow in the porous media and uses averages with several pore-widths
Quassi-geostrophic equations	It is applicable for rotating systems where it assumes a perfect balance between pressure-gradients and the coriolis and It is helpful in the atmospheric dynamics study.
Bernoulli's equation	It os the statement of energy conservation to solve non-viscous, incompressible fluid in steady flow where the sum of pressure and kinetic and potential energy per unit volume is constant at any point.
Darcy-Weisbach Equation	Due to the friction in pipes and ducts major Pressure loss happens.
Euler's equation	It governs the compressible and in viscid fluid with Navier Stoke equation of zero viscosity. It emphasis on the conservation of mass, momentum and energy.
Laplace Equation	It focuses on the behavior of gravitational, electric and fluid potential.
Ideal gas law	Change in density is directly related to the change in temperature and pressure.
Navier Stoke equation	It can be used to model a turbulent flow where the fluid parameters are interpreted as time-averaged values.
Mechanical energy equation	It is based on energy per unit mass., energy per unit volume and energy per unit weight involves Heads

VI. CONCLUSION

The above discussion analyses the types of fluid flows and the mathematical terms associated with the fluid flows. The numerical concepts of fluid flows are summarized with the efficacy of the equation models and are discussed. The parameters associated with different flows are summarized based on the literature survey on journals of fluid flows.

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