Performance Analysis Of Fractional Order Terminal SMC For The Half Car Model With Sinusoidal Road Input

S.Rajendiran*
Assistant Professor, Department of EIE,
Pondicherry Engineering College,
Puducherry, India
pecraja@gmail.com
*corresponding author

P.Lakshmi
Professor, Department of EEE
CEG, Anna University,
Chennai, India
p_lakshmi@annauniv.edu

***Abstract*—The Active Suspension System (ASS) in a vehicle helps to reduce the body acceleration and improves the ride. It produces a control force which is acting against the body acceleration. In this paper, a half car model with ASS is considered for analysis. the Fractional Order Terminal Sliding Mode Controller (FOTSMC) is suggested and its performance is compared with other types of SMCs in simulation. The FOTSMC's capacity to reduce the body acceleration is tested with sinusoidal road input to the HCM. From the output plots, Root Mean Square (RMS) and frequency weighted RMS, FOTSMC reduces the body acceleration in a better way than the other SMCs.**

***Index Terms*—Active suspension system, Fractional order terminal sliding mode control, Terminal sliding mode control, Half car model, Vibration control,sinusoidal road**

I. Introduction

The vertical vibration plays a major role in the analysis of travel comfort of a vehicle, therefore considering the QCM [1], [2] is a common practice among the researchers. After the controllers are tested with the simple QCM, the complexity of the system should go the next level, therefore considering the Half Car Model (HCM) is a common practice among the researchers [3], [4].

The reviewes of QCM and HCM with control strategies are presented in [5]. The different kinds of estimation [6] and optimization [7] techniques are used in PSS of the HCM. The optimization is based on the road input [8]. The performance of the optimized and normal values are compared and shows the effectiveness of the optimization techniques. The SASS used for the HCM [9] is changing the coefficient of magneto-rheological damper by means of decentralized neuro-fuzzy logic control [10]

The control strategies of the HCM with adaptive back stepping control [11], robust H infinity control [12], reliability control [13], and intelligent control [4] are proposed and the performances were analysed. The chattering free Terminal Sliding Mode Controller (TSMC) [14] is designed and applied to the nonlinear fractional-order dynamical system. It proves that the TSMC is useful for the robust stabilization of large class uncertainties of the fractional-order dynamical systems.

FOTSMC is applied to the quarter car with driver model to enhance the travel comfort [15]. The ride comfort [16] is analysed based on the frequency weighted root mean square values as defined by ISO 2631. In this paper, FOTSMC is proposed to reduce the BA of the HCM for sinusoidal road. The performance of the FOTSMC is compared with other designed SMCs.

II. MATHEMATICAL MODELLING OF THE HALF CAR

A typical HCM is shown in Figure 1. It consists of the sprung mass (m_s) represents the chassis of the car. The unsprung mass of the front (m_{uf}) and rear (m_{ur}) are representing the two wheel axle parts. Two actuators are mounted in the front and rear suspension of the HCM [17]. The force produced by the front actuator is F_{af} and the rear actuator is F_{ar}. The subscripts s, u, r, f, t and i represents sprung, unsprung, rear, front, tyre and input from road respectively. The vertical movement of the sprung mass (z_s) is considered at centre of m_s. θ is the pitch angle. L_f and L_r are the length of front and rear tyre from centre of m_s respectively.

In HCM, the front and rear wheels of the vehicle are connected in sprung mass with mechanical linkages. Hence,

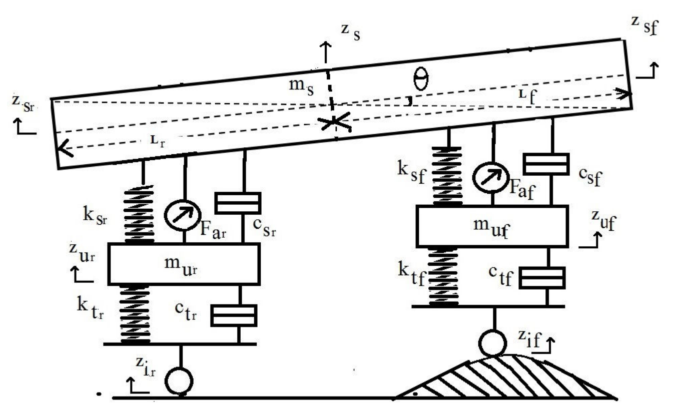

Fig. 1. Half car model

the interaction between front wheel and rear wheel is observed by means of the pitch angle and measured at the centre of the sprung mass. Therefore, the analysis of the vibration control in HCM is better than QCM. The HCM model process all the characteristics of the QCM and additional characteristics (i.e PA) from the full car model. Though the effect of road disturbance in rear wheel to front portion of sprung mass is less, it should be considered to produce better control force. Therefore, the control force applied to the front actuator is based on road input of front wheel as well as rear wheel and vice-verse.

$$\ddot{z}_{ur} = \frac{1}{m_{ur}}[-k_{tr}(z_{ur} - z_{ir}) - c_{tr}(\dot{z}_{ur} - \dot{z}_{ir}) + k_{sr}(z_{ur} - z_{sr}) + c_{sr}(\dot{z}_{ur} - \dot{z}_{sr}) - F_{ar}] \quad (1)$$

$$\ddot{z}_{uf} = \frac{1}{m_{uf}}[-k_{if}(z_{uf} - z_{tf}) - c_{if}(\dot{z}_{uf} - \dot{z}_{tf}) + k_{sf}(z_{uf} - z_{sf}) + c_{sf}(\dot{z}_{uf} - \dot{z}_{sf}) - F_{af}] \quad (2)$$

$$\ddot{z}_s = \frac{1}{m_s}[-k_{sf}(z_{sf} - z_{uf}) - c_{sf}(\dot{z}_{sf} - \dot{z}_{uf}) - k_{sr}(z_{sr} - z_{ur}) - c_{sr}(\dot{z}_{sr} - \dot{z}_{ur}) + F_{af} + F_{ar}] \quad (3)$$

$$\ddot{\theta} = \frac{1}{I_p}[-L_r(k_{sr}(z_{sr} - z_{ur}) + c_{sr}(\dot{z}_{sr} - \dot{z}_{ur})) + L_f(k_{sf}(z_{sf} - z_{uf}) + c_{sf}(\dot{z}_{sf} - \dot{z}_{uf})) - L_f F_{af} + L_r F_{ar}] \quad (4)$$

where I_p is the moment of inertia. The displacement of the front and rear sprung mass can be expressed in terms of pitch angle and sprung mass displacement as follows

$$z_{sf} = z_s - L_f\theta \quad (5)$$

$$\ddot{z}_{sf} = \ddot{z}_s - L_f\ddot{\theta} \quad (6)$$

by substituting the $\ddot{z}_s$ and $\ddot{\theta}$,

$$\ddot{z}_{sf} = a_1[F_{af} - k_{sf}(z_{sf} - z_{uf}) - c_{sf}(\dot{z}_{sf} - \dot{z}_{uf})] + a_2[F_{ar} - k_{sr}(z_{sr} - z_{ur}) - c_{sr}(\dot{z}_{sr} - \dot{z}_{ur})] \quad (7)$$

Similarly,

$$z_{sr} = z_s + L_r\theta \quad (8)$$

$$\ddot{z}_{sr} = \ddot{z}_s + L_r\ddot{\theta} \quad (9)$$

$$\ddot{z}_{sr} = a_2[F_{af} - k_{sf}(z_{sf} - z_{uf}) - c_{sf}(\dot{z}_{sf} - \dot{z}_{uf})] + a_3[F_{ar} - k_{sr}(z_{sr} - z_{ur}) - c_{sr}(\dot{z}_{sr} - \dot{z}_{ur})] \quad (10)$$

where $a_1 = \frac{1}{m_s} + \frac{L_f^2}{I_p}$, $a_2 = \frac{1}{m_s} - \frac{L_f L_r}{I_p}$, and $a_3 = \frac{1}{m_s} + \frac{L_r^2}{I_p}$

The FOTSMC, FOSMC and TSMC are designed for the HCM. The procedure followed to design the SMCs for the QCM is used to design the SMCs for the HCM.
To design F_{af} the state variables are chosen as follows,
front suspension deflection

$$x_1 = z_{sf} - z_{uf} \quad (11)$$

and front car body velocity is

$$x_2 = \dot{z}_{sf} \quad (12)$$

Similarly, to design F_{ar}, the state variables are chosen as follows
rear suspension deflection

$$x_1 = z_{sr} - z_{ur} \quad (13)$$

rear car body velocity

$$x_2 = \dot{z}_{sr} \quad (14)$$

A. Fractional order sliding mode controller

The FOSMC is designed as per the detailed producer given in [18] To design the control force f_a, the fractional order sliding surface is chosen as,

$$s = x_2 + \lambda x_1 \quad (15)$$

by differentiation,

$$\dot{s} = D^\alpha x_2 + \lambda \dot{x}_1 \quad (16)$$

The control forces in FOSMC is designed as

$$F_{af} = [k_{sf}(z_{sf} - z_{uf}) + c_{sf}(\dot{z}_{sf} - \dot{z}_{uf})] - \frac{a_2}{a_1}[F_{ar} - k_{sr}(z_{sr} - z_{ur}) - c_{sr}(\dot{z}_{sr} - \dot{z}_{ur})] - \frac{D^{1-\alpha}}{a_1}\lambda(\dot{z}_{sf} - \dot{z}_{uf}) - k\ sign(s) \quad (17)$$

$$F_{ar} = -\frac{a_2}{a_3}[F_{af} - k_{sf}(z_{sf} - z_{uf}) - c_{sf}(\dot{z}_{sf} - \dot{z}_{uf})] - [-k_{sr}(z_{sr} - z_{ur}) - c_{sr}(\dot{z}_{sr} - \dot{z}_{ur})] - \frac{D^{1-\alpha}}{a_3}\lambda(\dot{z}_{sr} - \dot{z}_{ur}) - k\ sign(s) \quad (18)$$

B. Terminal sliding mode controller

The TSMC is designed as per the detailed producer given in [18] To design TSMC, the terminal sliding surface is chosen as

$$s = (x_2)^{\frac{p}{q}} + \lambda x_1 \quad (19)$$

where p and q are integers which satisfy the condition $1 < \frac{p}{q} < 2$ [19]. Taking derivative on both sides

$$\dot{s} = \frac{p}{q}(x_2)^{\frac{p}{q}-1}\dot{x}_2 + \lambda \dot{x}_1 \quad (20)$$

$$F_{af} = [k_{sf}(z_{sf} - z_{uf}) + c_{sf}(\dot{z}_{sf} - \dot{z}_{uf})] - \frac{a_2}{a_1}[F_{ar} - k_{sr}(z_{sr} - z_{ur}) - c_{sr}(\dot{z}_{sr} - \dot{z}_{ur})] - (\frac{q}{p})(\frac{\lambda}{a1})(\dot{z}_{sf} - \dot{z}_{uf})^{(}2 - \frac{p}{q}) - k\ sign(s) \quad (21)$$

$$F_{ar} = -\frac{a_2}{a_3}[F_{af} - k_{sf}(z_{sf} - z_{uf}) - c_{sf}(\dot{z}_{sf} - \dot{z}_{uf})] - [-k_{sr}(z_{sr} - z_{ur}) - c_{sr}(\dot{z}_{sr} - \dot{z}_{ur})] - (\frac{\lambda}{a_3})(\frac{q}{p})(\dot{z}_{sr} - \dot{z}_{ur})^{(2-\frac{p}{q})} - k\ sign(s) \quad (22)$$

C. Fractional order terminal sliding mode controller

The FOTSMC is designed as per the detailed producer given in [18]

$$s = (D^{\alpha} x_2)^{\frac{p}{q}} + \lambda x_1 \tag{23}$$

$$\begin{aligned} F_{af} &= [k_{sf}(z_{sf} - z_{uf}) + c_{sf}(\dot{z}_{sf} - \dot{z}_{uf})] \\ &-\frac{a_2}{a_1}[F_{ar} - k_{sr}(z_{sr} - z_{ur}) - c_{sr}(\dot{z}_{sr} - \dot{z}_{ur})]- \\ &(\frac{q}{p})(\frac{\lambda}{a_1})(D^{1-\alpha})^{\frac{p}{q}}(\dot{z}_{sf} - \dot{z}_{uf})^{2-\frac{p}{q}} - k\ sign(s) \end{aligned} \tag{24}$$

$$\begin{aligned} F_{ar} = -\frac{a_2}{a_3}&[F_{af} - k_{sf}(z_{sf} - z_{uf}) - c_{sf}(\dot{z}_{sf} - \dot{z}_{uf})] \\ &-[-k_{sr}(z_{sr} - z_{ur}) - c_{sr}(\dot{z}_{sr} - \dot{z}_{ur})] \\ &-(\frac{q}{p})(\frac{\lambda}{a_3})(D^{1-\alpha})^{\frac{p}{q}}(\dot{z}_{sr} - \dot{z}_{ur})^{2-\frac{p}{q}} - k\ sign(s) \end{aligned} \tag{25}$$

III. Simulation and Results

The HCM is simulated using SIMULINK blocks of MATLAB 2012b. The vehicle parameters [7]. The sprung mass acceleration (i.e. BA) of the HCM is controlled by the designed controller. Since the BA is measured at front and rear end of the ms, the Front Car BA (FCBA) and Rear Car BA (RCBA) are considered as the feedback for the controller s design. The interaction between the front and rear wheel is analysed with respect to the Pitch Acceleration (PA) of the HCM. In SMC the Fequ highly oscillating when the λ is zero. The larger value of the λ is not useful to reduce the BA. Therefore, λ is chosen by trial and error method and is fixed at 1. Similarly, k is fixed as 1. To compare the performances of the controllers, the common parameters such as λ and k are fixed as the same values for all the SMCs. The terminal parameters are chosen as p/q=1.4 for both TSMC and FOTSMC. The λ = 0.9 for both FOSMC and FOTSMC. The simulations are performed for the period of 2 seconds.

A. Analysis of SMCs with Sinusoidal Road

In a road surfaces the continuous speed breakers are placed to force the vehicle to reduce the speed. These speed breakers are modelled as the Sinusoidal Road (SR) input and the performance of the SMCs are tested. The Equation 26 shows that SR is modelled in terms of vehicle speed and length of the speed breaker [20].

$$z_r(t) = 0.1\ sin(2\pi f t) \tag{26}$$

where $f = \frac{v_0}{\lambda_a}$, λ_a is the wavelength (1.66m) and the vehicle speed is v_0 (10 kmph). Figure 2 shows the sinusoidal road. In this case, the road input has the positive and negative magnitudes therefore the passive response and the time response with the controllers has high in magnitude. The force produced by the controllers are also high compared with the SB.

The passive response of the FCBA and RCBA are shown in Figure 3. The response of the HCM with the controllers for FCBA and RCBA are shown in Figure 4 and Figure 5 respectively. The forces produced by the front actuator and the rear actuator are shown in Figure 6 and Figure 7 respectively. The Performance Index for the SR road input is computed and tabulated in Table I.

TABLE I
Performance Index of HCM-SR

FCBA				
Controller	**RMS of BA (m/s^2)**	**FWRMS of BA (m/s^2)**	**UL**	**F_{af} (N)**
Passive	7.170	3.615	EU	0
FOSMC	0.344	0.222	NU	1025.900
TSMC	0.537	0.362	LU	965.200
FOTSMC	**0.329**	**0.178**	NU	1033.300
RCBA				
Controller	**RMS of BA (m/s^2)**	**FWRMS of BA (m/s^2)**	**UL**	F_{ar} (N)
Passive	5.792	2.437	VU	0
FOSMC	0.327	0.651	FU	969.200
TSMC	0.554	0.692	FU	895.000
FOTSMC	**0.312**	**0.458**	FU	983.100

Where NU- Not Uncomfortable VU - Very Uncomfortable LU - Little Uncomfortable EU- Extremely Uncomfortable FU- Fairly Uncomfortable UL - Uncomfort Level

The FOTSMC reduces the FCBA by 71.44%, whereas TSMC and FOSMC reduce the FCBA by 59.04% and 75.61 % respectively. FOTSMC controls the FCBA 16.58 % better than the TSMC and 4.17% better than the FOSMC. The performance of the FOSMC is 12.4 % better than TMSC. Similarly, FOTSMC reduces the RCBA by 77.37%, whereas TSMC and FOSMC reduce the RCBA by 59.53% and 73.74% respectively. FOTSMC controls the RCBA 17.84% better than the TSMC and 3.63% better than the FOSMC. The performance of the FOSMC is 14.21 % better than TMSC.

Figure 8 and 9 shows the passive PA and its reduction by SMCs. The PA is reduced by the FOTSMC by 95.6% which is 2.59% better than TSMC and 0.29% better than FOSMC. Figure 10 compares the reduction of PA with respect to the RMS values. The ride comfort is of the passive system with respect to FWRMS is ‘Extremely uncomfortable’ and ‘Little uncomfortable’ for FCBA and RCBA respectively. In FCBA the ride comfort is enhanced to ‘Not uncomfortable’ and ‘Little uncomfortable’ by FOSMC, FOTSMC and TSMC respectively. In FCBA the ride comfort is improved to ‘Fairly uncomfortable’ by the SMCs.

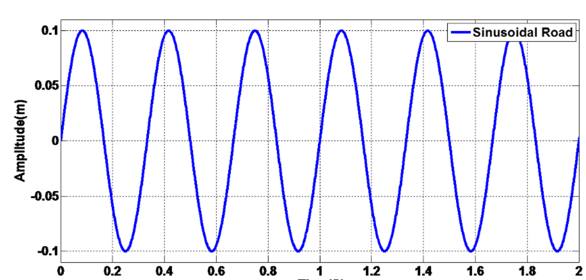

Fig. 2. Sinusoidal road

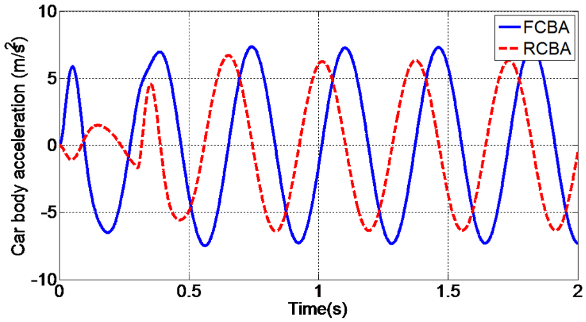

Fig. 3. Passive response of the HCM-SR

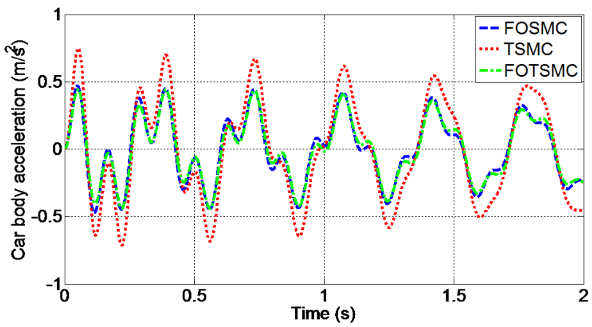

Fig. 4. FCBA of HCM with SMCs-SR

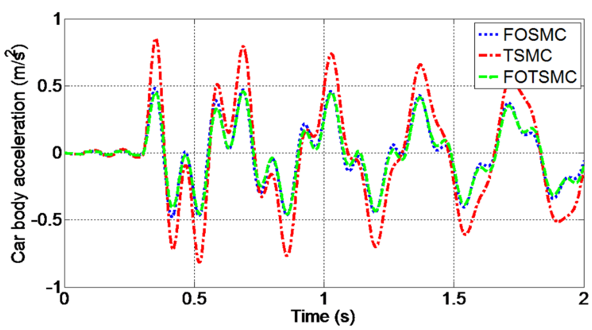

Fig. 5. RCBA of HCM with SMCs-SR

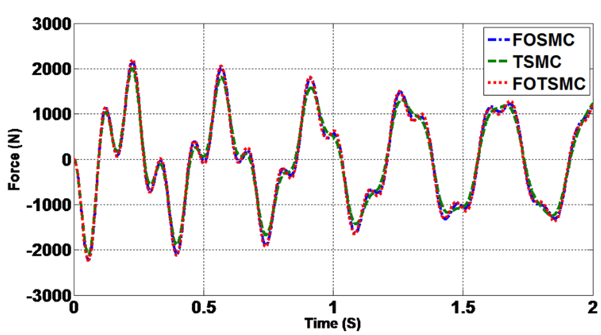

Fig. 6. Force Produced by the front actuator-SR

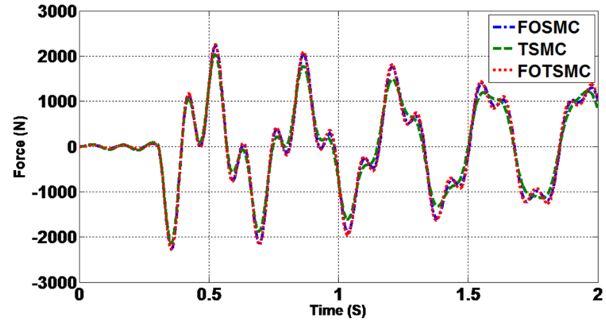

Fig. 7. Force Produced by the rear actuator-SR

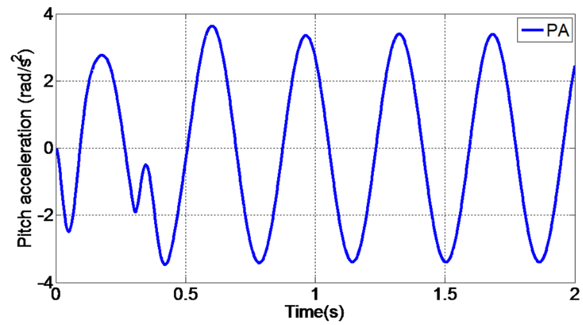

Fig. 8. Passive PA of the HCM-SR

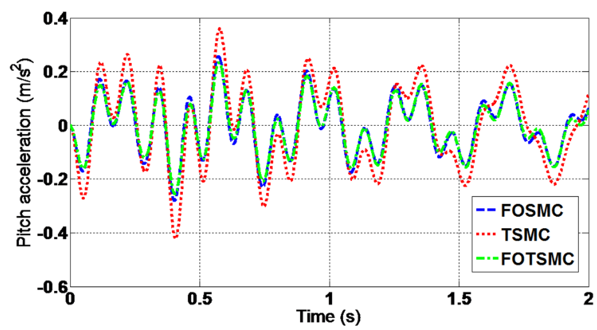

Fig. 9. PA of HCM with SMCs-SR

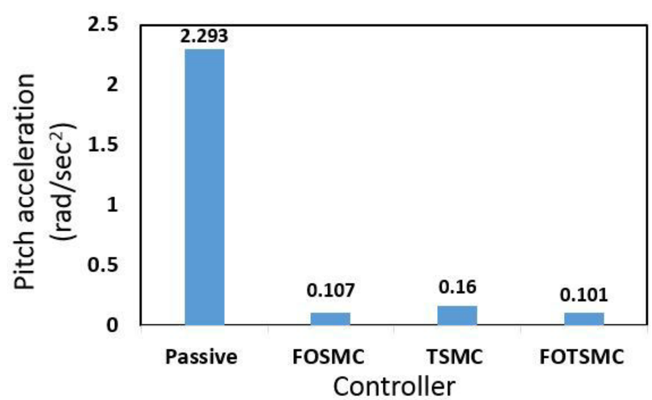

Fig. 10. Comparison PA in HCM-SR

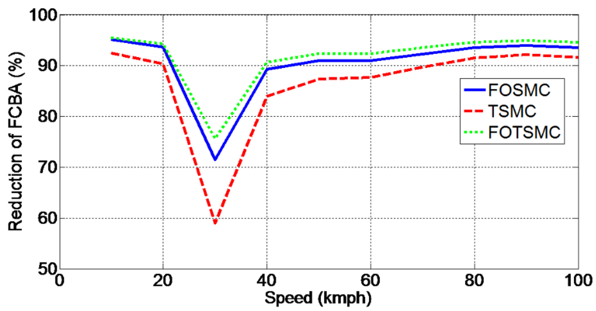

Fig. 11. Performance of SMCs in FCBA at different speed-SR

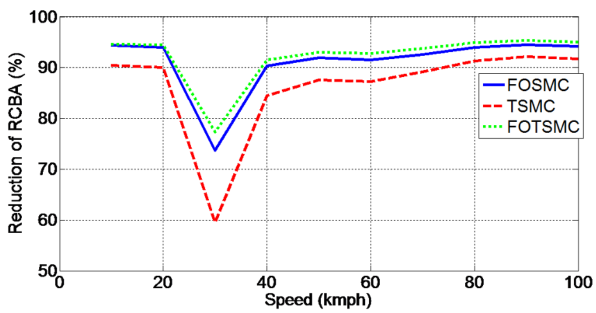

Fig. 12. Performance of SMCs in RCBA at different speed-SR

As the speed increase, the performance decreases gradually. Figure 11 and 12 shows the reduction of performances of the SMC with respect to speed. When the speed is 30 kmph, the system frequency and sinusoidal road inputs are equal therefore, HCM is vibrating maximum value at resonance. The passive response produced by the HCM for the FCBA as 29.661 m/s^2 and RCBA as 30.629 m/s^2. Therefore the SMCs are performance are low at 30 kmph. The continuous road disturbance in SR is a challenge for the controllers to produce the higher force quickly.

The PSD for the FCBA and RCBA are plotted for RCBA and FCBA (Figure 13 to Figure 14). FOTSMC reduced BA successfully in the human sensitive frequency ranges.

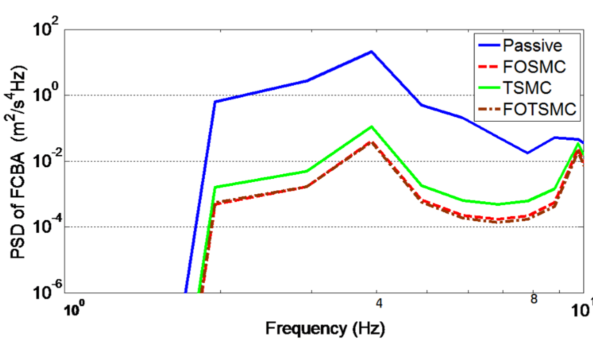

Fig. 13. PSD of FCBA-SR

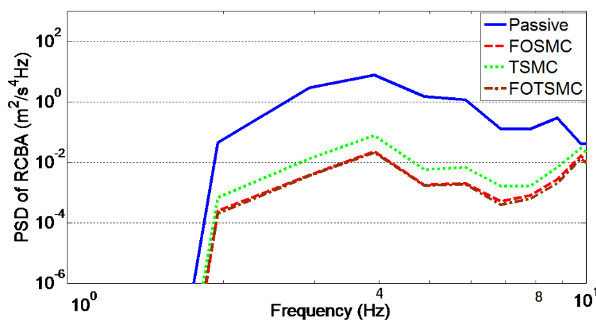

Fig. 14. PSD of RCBA-SR

IV. Conclusion

The FOTSMC, TSMC, FOSMCs are designed and simulated for the HCM with Sinusoidal road. The simulated controllers reduces the front car body acceleration, rear car body acceleration and pitch acceleration of the HCM. The performance of the FOTSMC are compared against other SMCs. FOTSMC reduces BAs and PA better than other SMCs with hihger control force. When the speed of the vehicle is increased, the performance of SMCs are decreased. The ride quality is improved by the designed SMCs as per the ISO 2631.

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