Detour Distance Sequence of Grid and Grid Derived Networks

S. Prabhu[1*], Y. Sherlin Nisha[2], K.S. Sudhakhar[3]
[1]Assistant Professor, Department of Applied Mathematics
Sri Venkateswara College of Engineering (Autonomous), Sriperumbudur, India 602117.
[2]Assistant Professor, Department of Mathematics
Sri Sairam Institute of Technology, Chennai, India 600044.
[3]Assistant Professor, Department of Mathematics
St. Joseph's College of Engineering, Chennai, India 600119.
*Corresponding Author: drprabhu@svce.ac.in

***Abstract* – Let $G(V,E)$ a simple graph. Let $u,v \in V(G)$. The detour distance, $D(u,v)$, between u and v is the distance of the longest path from u to v. The detour graph of G, denoted by $D(G)$ is defined as an edge labeled complete graph on n vertices where $n = |V(G)|$, the labels for uv is $D(u,v)$. Detour Distance Sequence is a Sequence obtained by arranging the edge labels of the Detour graph in descending order. In this paper we find the Detour Distance Sequence of grid and its derived networks.**

***Keywords:* Detour, maximum distance, grid networks, delay time.**

I. Introduction

In graph theory most of them are familier in shortest path problem or shortest distance problem. But there is also another type of distance problem called maximum distance problem. In literature $d(x,y)$ is the length of the *shortest path* between nodes x and y. Similarly we define $D(x,y)$ is the length of the *longest path* between nodes x and y. This problem may be helpful to find the time bound to confirm whether the given network is faulty. This paper deals with the maximum distance between two nodes of the grid and grid derived archtectures. It is also a kind of edge labeling, using the labels of the edge we form the distance sequence.

II. An Overview of the Paper

E. Sampath Kumar et al. [16] introduces the detour graph of a given graph G, denoted by Let $G(V,E)$a simple graph. Let $u,v \in V(G)$. The detour distance, $D(u,v)$, between u and v is the distance of the longest path from u to v. The detour graph of G, denoted by $D(G)$ is defined as an edge labeled complete graph on n vertices where $n = |V(G)|$, the labels for uv is $D(u,v)$. Let $G = (V,E)$ be a simple graph. The detour graph of G is denoted by $D(G)$, is an edge labeled complete graph K_m, where $m = n[V(G)]$ such that for any two vertices a, b in G will be adjacent in $D(G)$ whose edge label is k if it is the length of longest path between a to b. This length is also called Detour distance. Detour Distance Sequence is a Sequence obtained by arranging the edge labels of the Detour graph in descending order. Every Detour graph will have its unique DDS (Detour Distance Sequence). For other problems on labelling refer [5].

If a detour graph of G has all the labels 1, then $G = K_2$. If a detour graph of G has all the labels 2, then $G = K_3$. The maximum distinct integers one can get in a detour distance sequence is n-1. The graph G is a tree if and only if the detour graph $D(G)$ contains (n-1) 1's. A Graph G is path if and only if the detour distance sequence is $1^{n-1},2^{n-2},\ldots,(n-1)^1$. A graph G is a star if and only if the detour distance sequence of G is $1^{n-1}, 2^{{}^nC_2-(n-1)}$.

Indra et al. [13] shown that $DDS(K_{m,n}) = (2m-2)^{{}^mC_2}, (2m-1)^{mn}, (2m)^{{}^nC_2}$ for $n > m$ where 'm' denotes the number of vertices which has the greater degree and 'n' denotes the number of vertices which has the lesser degree. $DDS(K_{n,n}), n \geq 2$ is $(2n-2)^{n(n-1)}, (2n-1)^{n^2}$. For other Detour distance related problems the reader can refer [2-4,7-12,15,17].

III. Detour Graph

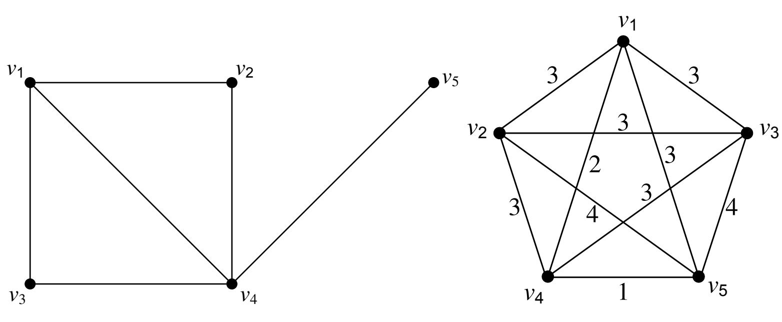


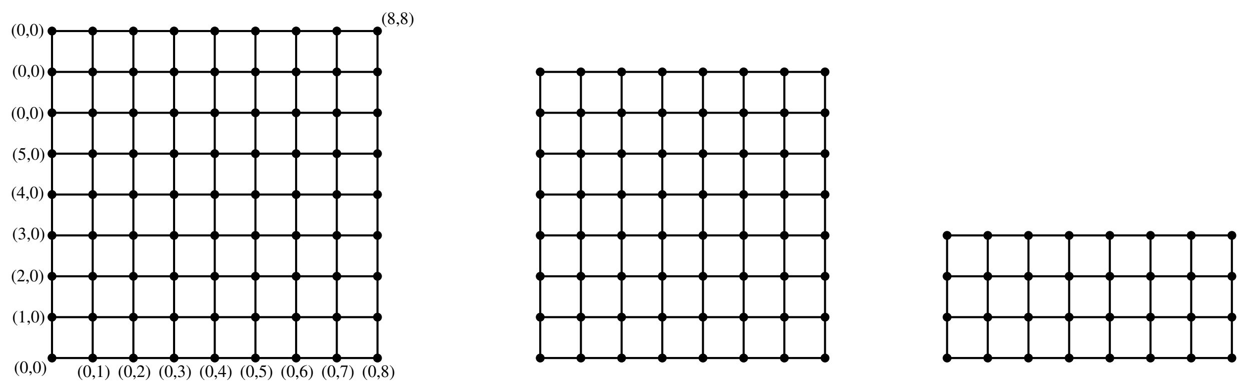
Fig 1. A Graph and its Detour Graph.

Let $G = (V,E)$ be a simple graph. For notations the reader can refer [1,6]. The detour graph of G is denoted by $D(G)$, is an edge labeled complete graph K_m, where $m= n[V(G)]$ such that for any two vertices a, b in G will be adjacent in $D(G)$ whose edge label is k if it is the length of longest path between a to b. This length is also called Detour distance.

IV. Grid Networks

A straight forward generalization of the linear (1-D) array is the Grid (2-D) array [14]. It is observed that the square Grid has diameter $2n$ - 2; vertex degree 2 or 4; almost symmetric. Grid Network Topology is one of the key network architectures in which devices are connected with many redundant interconnections between network nodes such as routers and switches.

A two-dimensional rectangular grid graph is an $m \times n$ graph $G(m, n)$, that is the Cartesian product of path graphs on m and n vertices respectively. If $m = n$ then the grid turns to a square grid. If $m \neq n$we say it as rectangle grid. We apply our problem on square grid, $G(m,m)$ See Figure 2.



$G(9,9)$ $G(8,8)$ $G(4,8)$

Fig 2. Grid Networks

Lemma 1: Let $G = G(m,m)$ and let $x = (0,0)$, $y \in N_r(x)$, Then $D(x,y) = \begin{cases} m^2 - 1, & r \ odd \\ m^2 - 2, & r \ even \end{cases}$, $1 \leq r \leq 2(m-1)$.

Theorem 4.1: $DDS\big(G(m,m)\big) = (m^2-2)^{\binom{m^2}{2}-\left(\frac{m^2}{2}\right)^2}, (m^2-1)^{\left(\frac{m^2}{2}\right)^2}$, where '$m$' is even.

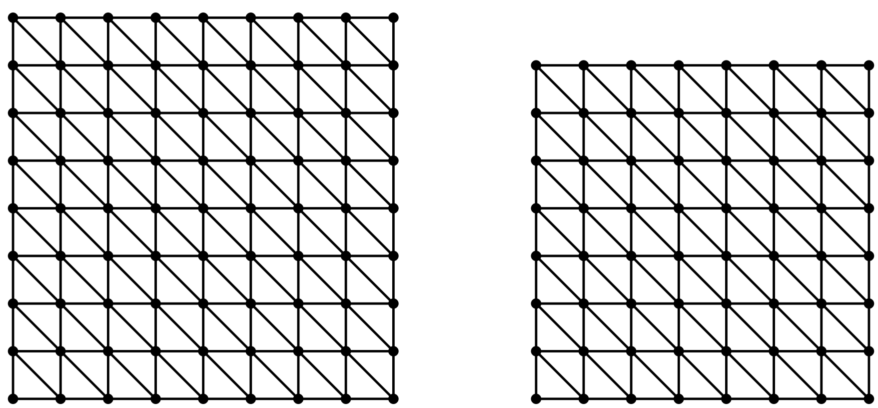
Proof. Let $G = G(m,m)$. First we find the Hamilton cycle between any two adjacent vertices of G. Let $x = (0,0)$ and $y = (0,1)$, by definition x and y are adjacent. Now we try to find another path (longest path) containing all remaining vertices of G. Let $P = [x,(1,0)] \circ P_i \circ [(m-1, i-1),(m-1,i)] \circ P_{i+1} \circ [(1,i),(1,i+1)] \circ [(1,m-1),(0,m-1)] \circ Q$, $P_i = (j, i-1)$, $P_{i+1} =$

$(m-1,i)$ and $Q=(0,m-j)$, where $1 \le i \le m-1$, i odd and $1 \le j \le m-1$. Similarly we can construct the one Hamilton path for the other non-adjacent pair of vertices which are at even distance. This proves that the DDS of this graph has only two components.

Theorem 4.2: For 'm' is odd.

$DDS\big(G(m,m)\big) = (m^2-3)^{\binom{m^2}{2}-3(m^3-2m^2+m-2)}$, $(m^2-2)^{2(m^3-2m^2+m-2)}$,$(m^2-1)^{(m^3-2m^2+m-2)}$.

V. AUGMENTED GRID NETWORK



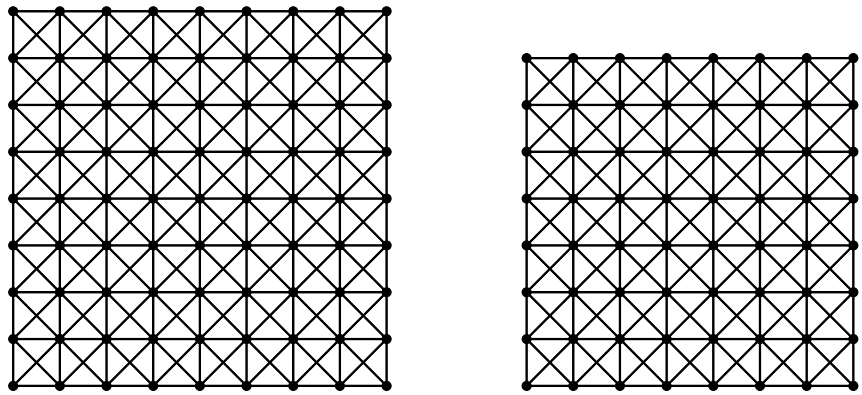
$AG(9,9)$ $AG(9,9)$

Fig 3. Augmented Grid Networks

Theorem 4.1: $DDS\big(AM(m,m)\big) = (m^2-1)^{\binom{m^2}{2}}$, where '$m$' is even.

VI. EXTENDED GRID NETWORK

Extended grid is derived from the standard $m \times n$ grid by making each 4-cycle is an $m \times n$ grid into a complete graph and denoted by $EX(m,n)$.



$EX(9,9)$ $EX(8,8)$

Fig 4. Extended Grid Networks

Theorem 4.1: $DDS\big(EX(m,m)\big) = (m^2-1)^{\binom{m^2}{2}}$, where '$m$' is even.

VII. CONCLUSION

In this paper we have derived the detour distance sequence of grid and its derived network. This distance sequnce the maximum may be considered as the time bound to test whether the given network is faulty. The frequency of the distance parameter in the distance sequence may be associated with the probability. This probability may be helpful in discussing faultness of the network. This problem is under investigation for other networks like hypercube and butterfly networks.

REFERENCES

[1] J.A. Bondy and U.S.R. Murthy, Graph Theory with Applications, The Macmillan Press Ltd. 1976.

[2] G.Chartrand, G.L. Johns and S. Tian, Detour Distance in Grpahs, Annals of Discrete Mathematics, 55 (1993) 127–136.

[3] M.V. Diudea, G. Katona, I. Lukovits, N. Trinajsti´c, Detour and Cluj-detour indices, Croat. Chem. Acta, 71 (1998) 459-471.

[4] C. Du, Minimum Detour Index of Bicyclic Graphs, MATCH Commun. Math. Comput. Chem. 68 (2012) 357–370.

[5] J.A. Gallian,"Dynamic Survey DS6: Graph Labeling", Electronic J. combinatorics. DS6, 1-58, 2007.

[6] F. Harary, Graph Theory, Addison-wesley Publishing Company, 1969.

[7] I.Lukovits, The Detour Index, Croat. Chem. Acta, 69 (1996) 873–882.

[8] Z. Mihalic, D. Veljan, D. Amic, S. Nikolic, D.Plavsic, N. Trinajstic, The Distance Matrix in Chemistry, J. Math. Chem. 11 (1992) 223–258.

[9] S.Prabhu, J. Praveen, On the Detour Index of Circular Ladder, *Int. J. Adv. Res. Basic Engg. Sci. and Tech.*, 3(12) (2017) 30-35.

[10] S.Prabhu, G. Murugan, Natarajan Muthuraman, On Degree-Distance Index of Hexagonal Network, *Int. J. Adv. Res. Basic Engg. Sci. and Tech.*, 4(3) (2018) 24-33.

[11] S.Prabhu, M. Arulperumjothi, G. Murugan, On Certain Topological Indices of Titanium dioxide Nanosheet and Nanotube, *Nanoscience & Nanotechnology-Asia*, 8 (2018) 309-316.

[12] S.Prabhu and M. Arulperumjothi, On Certain Topological Indices of Benzenoid Compounds, *Journal of Advances in Chemistry*, 13(8) (2017) 6406-6412.

[13] I.Rajasingh, B. Rajan, S. Prabhu, Maximum Distance Matrix of the Complete Bipartite Graph and its Distance Sequence, ICMCS 2009, pages 148-153.

[14] B.Rajan, I. Rajasingh, S.K. Thomas, and C. Monica, One factor resolving set of Grid-derived architectures, Proceedings of ICMCS, pp. 1-4, 2011.

[15] G.Rucker, C. Rucker, Symmetry-aided computation of the detour matrix and the detour index, J.Chem. Inf. Comput. Sci. 38 (1998) 710–714.

[16] E.Sampathkumar, V. Swaminathan, P. Vishvanathan, G. Prabakaran, Detour Graphs,ICMCS, Pages 247 & 248, 2006.

[17] N.Trinajstic, S. Nikolic, B. Lucic, The Detour Matrix in Chemistry, J. Chem. Inf. Comput. Sci. 1997, 37, 631-638.

AUTHOR(S) BIOGRAPHY

Dr. S. Prabhu is currently working as Assistant Professor of Department of Applied Mathematics, Sri Venkateswara College of Engineering (Autonomous), Sriperumbudur, India He Received his B.Sc., M.Sc., M.Phil. and Ph.D Form Loyola College (Autonomous), Chennai Affiliated to Madras University. During his B.Sc Course he received Rev. Fr. Alfrod Solomon Memorial Award for Securing highest Marks in the Major. During his Research Period, He received Rev. Fr. Racine Research Award. His Ph.D was Fully Sponsored by the Major Research Project No. F. 38-120/2009(SR) of the University Grants Commission, New Delhi, India. He also received the UNESCO-HP BGI Award given by Kuwait University Kuwait. He is a recognized supervisor of Anna University. Currently Five Students are doing Ph.D under his guidance.

Y.SHERLIN NISHA is currently working as Assistant Professor of Department of Mathematics, Sri Sairam Institute of Technology, Chennai, India. I have received my B.Sc., M.Sc. degree from Nesamony Memorial Christian College and M.Phil., from Scott Christian College, Nagercoil affiliated to Manonmaniam Sundaranar University. Currently I am doing Ph.D in Mathematics, Anna University under the guidance of Dr. S. Prabhu, Assistant Professor, Department of Applied Mathematics, Sri Venkateswara College of Engineering (Autonomous), Sriperumbudur, Chennai

Mr. K.S SUDHAKHAR is currently working as an Assistant Professor, Department of Mathematics, St. Joseph's College of Engineering, Chennai, India. He received his B.Sc., M.Sc., M. Phil., M.Ed., from University of Madras, Chennai. Currently he is perusing his Ph. D in Mathematics at University of Madras.