Strong Converging Detonation Waves in Rotating Ideal Gas

P.K. Gangwar[1], R.K. Verma[2]
Assistant Professor, Department of Physics, Bareilly College, Bareilly-243001, India [1]
Assistant Professor, Department of Physics, K.S. Saket (P.G.) College, Ayodhya-224123, India [2]

***Abstract*— The aim of this paper is to analyze the motion of strong converging spherical detonation waves in a reacting ideal gas having solid body rotation, by using Chester-Chisnell-Whitham (CCW) theory. It is assumed that detonation wave is initially Chapman-Jouguet. Initially taking the power varying initial density distribution, expressions for the detonation velocity and other flow variables are derived. Neglecting the effect of Coriolis forces, the effect of solid body rotation of reacting gas on detonation velocity and other flow variables has been estimated. The variation of detonation velocity, non-dimensional pressure and non-dimensional flow velocity with convergence of detonation front, density parameter, specific heat ratio of gas have been discussed though graphs in details. Finally, it is observed that density parameter, adiabatic index and Alfven Mach number of gas have significant effect on all the flow variables.**

***Keywords*: CCW method, Detonation waves, power varying density distribution. rotating ideal gas.**

I. INTRODUCTION

On account of its immense importance in Engineering Physics, several authors have tackled the problem of detonation wave propagation in different type of media. Welsh[1], Nigmatulin[2] and Teipel [3] have replaced the shock front by a contracting detonation front propagating into a uniform combustible ideal gas. In magnetogasdynamics, Vishwakarma and Vishwakarma[4] have extended the problem of Nigmatulin[2]. Verma and Singh[5], [6] has investigated the phenomenon of Tiepel[3] in non-homogeneous atmosphere. Chester-Chesnell-Witham (CCW) method[7], [8], [9] have been used by Tyl and Wlodarczyk[10] for theoretical investigation on the concentric detonation waves in gaseous explosive mixtures. Shock wave propagation in the rotating ideal and dusty gases have been presented by Gangwar[11], [12].

In the present work, the propagation of imploding detonation waves having spherical symmetry have been studied when the ideal gas having the solid body rotation. The CCW- method is used to solve the problem. The detonation is a Chapman-Jouguet front i.e. it. travels with sonic speed relative to the burst gas, which determines the law of convergence. The values of the pressure and internal energy in the undisturbed fluid have been neglected in comparison to their values in the disturbed gas i.e. strong detonation wave. The constant amount of heat is produced during the detonation process and by adding this, the basic flow equations only be corrected. [14] discussed that Marine energy resources, encompassing both wave and tidal energy, represent a vast and largely untapped renewable energy source. This paper explores the potential of marine energy to contribute to the global energy mix, the technological advancements that are facilitating its capture, and the challenges and opportunities associated with its development and integration into existing energy systems.

II. Fundamental Equations, Boundary Conditions and Analytical Expressions

The Fundamental equations for one-dimensional unsteady, adiabatic flow of an ideal reacting gas can be written as Tyle [13]

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\rho\frac{\partial p}{\partial r}-\frac{v^2}{r}=0 \tag{1}$$

$$\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial r}\right)\rho+\rho a^2\left(\frac{\partial u}{\partial t}+\frac{2u}{r}\right)=0 \tag{2}$$

$$\frac{\partial \varepsilon}{\partial t}+u\frac{\partial \varepsilon}{\partial r}+\frac{p}{\rho^2}\left(\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial r}\right)=0 \tag{3}$$

$$\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial r}\right)(vr)=0 \tag{4}$$

where u, p, ρ, v and ε are the particle velocity, the pressure, density, angular velocity of the medium, and internal energy per unit mass of the ideal gas and 'a' is the local speed of sound in ideal reacting gas is given by

$$a^2=\gamma p/\rho \tag{5}$$

where γ is the adiabatic index of the gas. The equation of state for ideal gas may be written as

$$p=\gamma\rho T \tag{6}$$

where $c_p-c_v=\gamma$ is the gas constant. c_p and c_v are the specific heats of the gas at constant pressure and volume, respectively.

The internal energy per unit mass of the ideal gas may be written as

$$\varepsilon=p/\rho(\gamma-1) \tag{7}$$

The density of the gas ahead of the imploding detonation front are assumed to be power varying and obeying the law:

$$\rho_a=A_i\eta^w \tag{8}$$

where A_i is the constant and the $\eta=r_a/R$ is the ratio of radius 'r_a' to the internal radius of the detonation front 'R' and 'w'is density parameter. The angular velocity 'v' may be given as

$$v = \eta \Omega_a \tag{9}$$

where Ω_a is the initial angular velocity which is taken constant in this study.

Let u_a, ρ_a, p_a and ε_a represent the undisturbed values of flow velocity, density, pressure, and internal energy per unit mass of ideal gas just ahead of the detonation front and u, ρ, p and ε be the modified values of respective quantities at any point across the passage of the detonation front. The jump conditions across the strong detonation in ideal gas detonation front in this case may be written as Whitham[9], Tyl and Wlodarczyk, Vishwakarma and Vishwakarma [4]

$$\rho\left(U_D - u\right) = \rho_a U_D \tag{10}$$

$$p = p_a + \rho_a U_D u_a \tag{11}$$

$$\varepsilon = \varepsilon_a + \frac{1}{2}\left(p + p_a\right)\left(\frac{1}{\rho_a} - \frac{1}{\rho}\right) + q \tag{12}$$

where U_D, and q denote the velocity of detonation front and heat energy released per unit mass, respectively. The indices 'a' refers to the sates just ahead the detonation front, respectively.

A. CCW solution of the problem

The detonation front is assumed to be in Chapman-Jouguet state. Chapman-Jouguet condition requires that the flow ahead of the shock front will be in sonic state and, in the shock-fixed coordinates, i.e.

$$\left|U_{D_{CJ}} - u_{CJ}\right| = a_{CJ} \tag{13}$$

where the indices 'CJ' denote the Chapman-Jouguet state.

The boundary conditions across the detonation front in this case are

$$p_{CJ} = \frac{\rho_a}{\gamma + 1} U_{D_{CJ}}^2 \tag{14}$$

$$u_{CJ} = \frac{1}{\gamma + 1} U_{D_{CJ}} \tag{15}$$

$$\rho_{CJ} = \frac{\gamma + 1}{\gamma} \rho_a \tag{16}$$

$$a_{CJ} = \frac{\gamma}{\gamma + 1}\left|U_{D_{CJ}}\right| \tag{17}$$

$$U_{D_{CJ}} = \sqrt{2\left(\gamma^2 - 1\right) q} \tag{18}$$

Using the equation (8) and equations (13)-(18) the boundary conditions across the strong detonation front having power varying density distribution may be written in the term of velocity of burnt gas

$$\frac{U_D}{U_{D_{CJ}}}=\frac{N^2-1}{2N} \tag{19}$$

$$\frac{p}{p_{CJ}}=\frac{N^2+1}{2}\eta^w \tag{20}$$

$$\frac{\rho}{\rho_{CJ}}=\frac{\gamma\left(N^2+1\right)}{\left(\gamma-1\right)N^2+\gamma+1}\eta^w \tag{21}$$

$$\frac{a}{a_{CJ}}=\frac{\left(\gamma-1\right)N^2+\gamma+1}{\sqrt{2\gamma\left[\left(\gamma+1\right)+\left(\gamma-1\right)N^2\right]}} \tag{22}$$

where $N=u_n/u_{CJ}$ and $\eta=r_a/R$, The detonation velocity in terms of Alfven Mach number under Chapman-Jouguet state may be given by [15]

$$U_D=U_{CJ} \text{ and } U_{D_{CJ}}=M_{CJ}a_0 \tag{23}$$

where $a_0^2=\gamma\, p_0/\rho_0$ and M_{CJ} is the Alfven Mach number. At the equilibrium state, of the gas is assumed to be specified by the condition, $u=0=\partial/\partial t$, $p=p_0$, and $v=v_0=r_0\Omega_0$, where v_0 is the azimuthal component velocity (rotational velocity) and Ω_0 is the initial angular velocity. Therefore, from equation(1), the equilibrium condition prevailing the front of the shock can be written as

$$\frac{1}{\rho_0}\frac{dp_0}{dr}=\frac{v_0^2}{r}=\eta\Omega_0^2 \tag{24}$$

On solving the above equation, we have

$$p_0=A_i\frac{\eta^{w+2}}{w+2}\Omega_0^2 \tag{25}$$

The local speed of sound in the ideal gas a_0 ahead of the detonation front, having solid body rotation with angular velocity w with power varying density distribution can be given by using the equations(5) and (25) we have:

$$a_0^2=\eta^2\Omega_0^2\frac{\gamma}{\left(w+2\right)} \tag{26}$$

Using the equations(15),(18) and (23), we get

$$u_{CJ}=\frac{M_{CJ}\Omega_0\eta}{\gamma+1}\left(\frac{\gamma}{w+2}\right)^{1/2} \tag{27}$$

The characteristic form of the governing equations for converging shock i. e. the form in which equation contains derivatives in only one direction in (r, t) plane is

$$dp - \rho a du + \frac{2\rho a^2 u}{(u-a)}\frac{dr}{r} + \frac{\rho a}{(u-a)}\frac{v^2 dr}{r} = 0 \tag{28}$$

Equation(28) divided by $\rho_{CJ} u_{CJ}^2$ and using the equations (8)(9),(19)-(27) after simplifying, we have

$$\frac{dN}{d\eta} = \frac{N^2+1}{\eta\left[\gamma\left(N^2+1\right)\beta - N\right]}\left[\frac{w}{2} + \frac{\beta\left(\alpha\left\{(\gamma-1)N^2+\gamma+1\right\}\beta N\gamma M_{CJ}^2 - (w+2)(\gamma+1)^2\right)}{\left[N-\gamma\left\{(\gamma-1)N^2+\gamma+1\right\}\beta\right]M_{CJ}^2}\right] \tag{29}$$

where $\beta = \left[2\gamma\left\{(\gamma+1)+(\gamma-1)N^2\right\}\right]^{-1/2}$

Numerical integrating the differential equation (29) and using equations(19)-(21), we get the variation of N, $\frac{U_D}{U_{D_{CJ}}}$, $\frac{p}{p_{CJ}}$ and $\frac{\rho}{\rho_{CJ}}$ with propagation distance η. .[10] presented a book, We know, correspondence implies exchange of data from source to beneficiary. In conventional communication, when source and beneficiary were situated in long separation, this exchange used to occur by interfacing source and beneficiary physically through leading wires, which would convey data as electrical signs.

III. Results and Discussion

Using the equations(19)-(21) and equation (28), the analytical expression for detonation front velocity $\frac{U_D}{U_{D_{CJ}}}$, the pressure behind the detonation front $\frac{p}{p_{CJ}}$, and the density across the front $\frac{\rho}{\rho_{CJ}}$ in terms of the non-dimensional particle velocity $\frac{u}{u_{CJ}}$ just behind the strong spherical converging detonation wave in a rotating ideal gaseous atmosphere are, respectively, have been calculated numerically. It represents that detonation front velocity $\frac{U_D}{U_{D_{CJ}}}$, the pressure behind the detonation front $\frac{p}{p_{CJ}}$, and the density across the front $\frac{\rho}{\rho_{CJ}}$ all flow parameters are depend upon the propagation distance $\left(\eta = \frac{r_0}{R}\right)$, specific heat ratio of the mixture(γ), density parameter(w), and Alfven Mach number M_{CJ}.

It is observed from the expressions all the flow parameters are not directly depends upon initial angular velocity Ω_0 as in the case of simple shock wave propagation under the effect of solid body rotation of gas. Taking initially the strength of freely propagating shock $\frac{u}{u_{CJ}} = 2$ and $\gamma = 2$. The variation of detonation front velocity $\frac{U_D}{U_{D_{CJ}}}$ with the propagation distance (η), the pressure behind the detonation front $\frac{p}{p_{CJ}}$, and the density across the front $\frac{\rho}{\rho_{CJ}}$ for $\gamma = 2,4; w = 1.5,2$ and $M_{CJ} = 5,10$ have been calculated and depicted through Fig. (1)-(3). It is observed from Fig.(1) that the strong detonation front velocity $\frac{U_D}{U_{D_{CJ}}}$ is decreases as shock converges in the medium having solid body rotation with power varying initial density distribution. As the value of density parameter w increases from 1.5 to 2 the slope of the graph (1)-(3) in increases at fix value M_{CJ}.

The variation of pressure behind the detonation front $\frac{p}{p_{CJ}}$, and the density across the front $\frac{\rho}{\rho_{CJ}}$ with propagation distance have been shown in the figure (2) and (3), respectively. It is noticed that the increase in Alfven Mach number M_{CJ} from 5 to 10 is reduces the pressure $\frac{p}{p_{CJ}}$ as well as density across the rotating strong convergent front of detonation wave. It is also observed that strength of detonation wave is also increase with density parameter (w). The impact of change in adiabatic index(γ) of the gas is also represents in the Fig.(1)-(3) for fix values of other parameters. It is found that all flow parameters $\frac{U_D}{U_{D_{CJ}}}, \frac{p}{p_{CJ}}$ and $\frac{\rho}{\rho_{CJ}}$ are decreases with increase in adiabatic index. Similar behavior on the adiabatic convergence of simple spherical shock propagation in an ideal gas having solid body rotation has been obtained by the use of CCW theory[16].

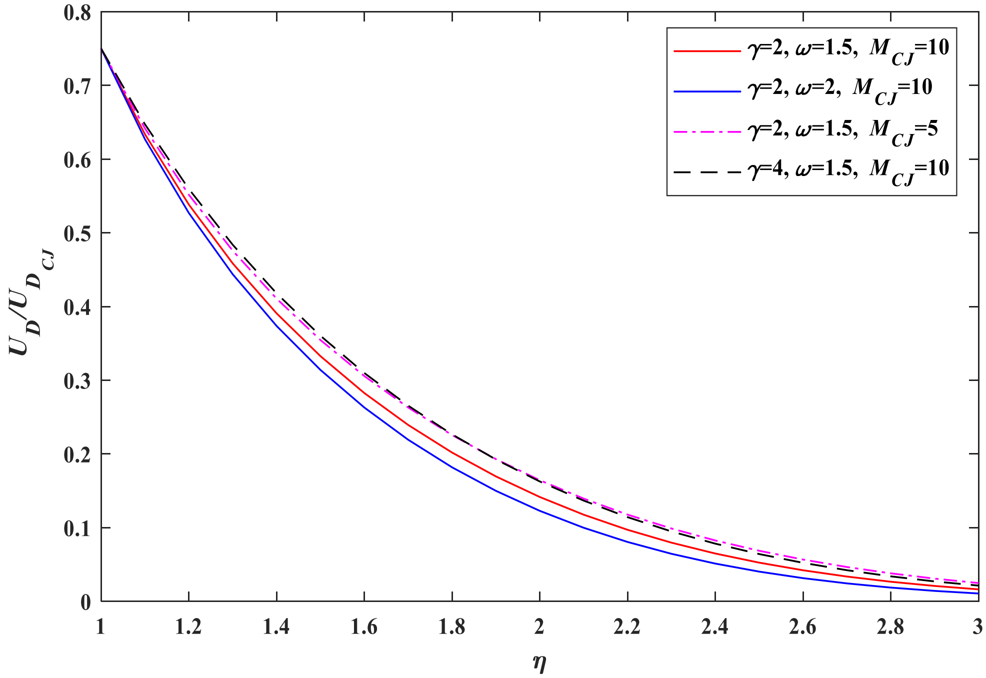

Fig. 1. Variation of detonation front velocity $\frac{U_D}{U_{D_{CJ}}}$ with the propagation distance (η)

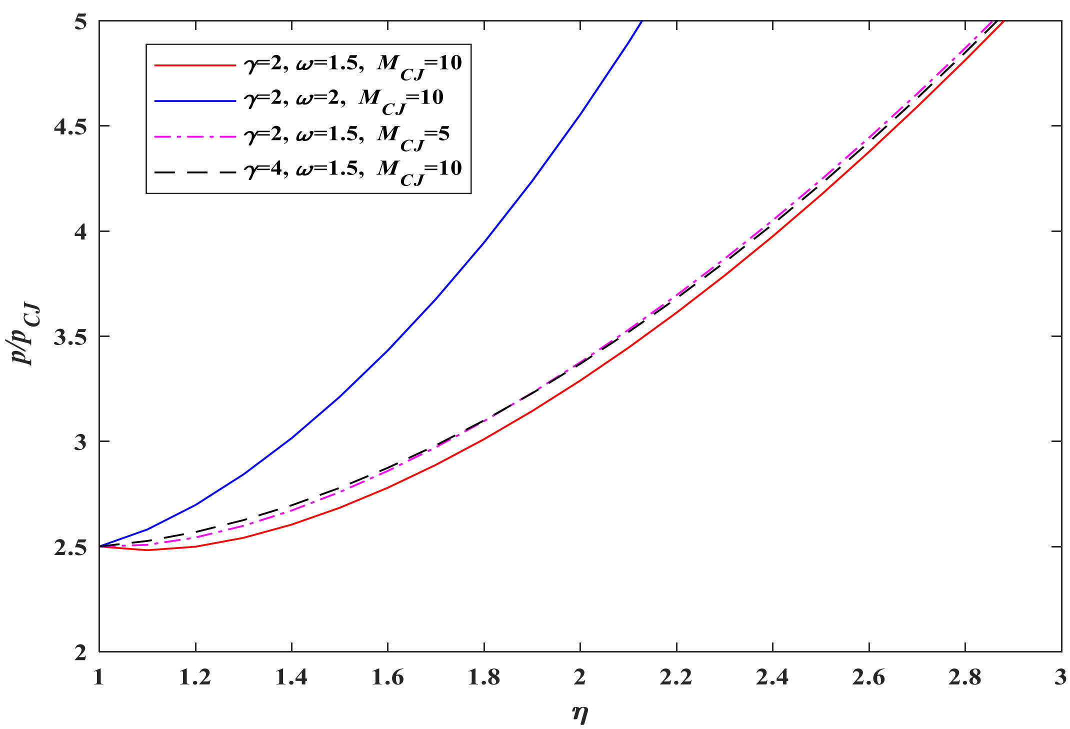

Fig. 2. Variation of pressure behind the detonation front $\frac{p}{p_{CJ}}$ with the propagation distance (η)

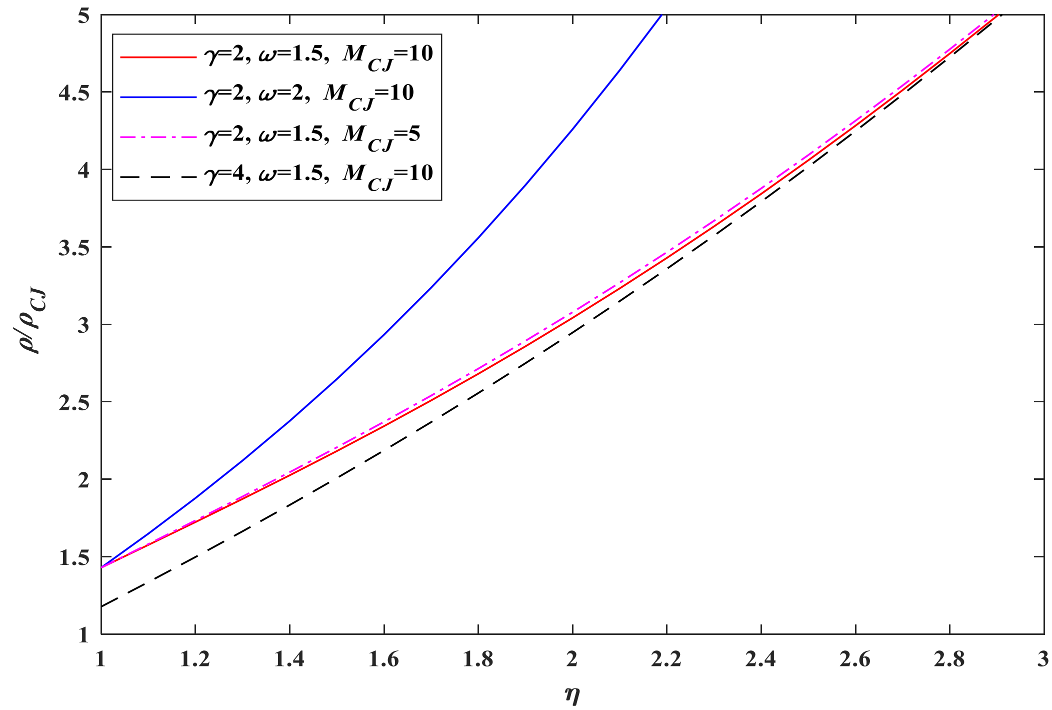

Fig. 3: Variation of density across the front $\frac{\rho}{\rho_{CJ}}$ with the propagation distance (η)

IV. Conclusion

The problem of spherical strong converging detonation wave propagation in ideal reacting gaseous medium having solid body rotation have been carried out by the method of CCW for the variable initial density distribution. The behavior of the ideal gas across the strong detonation front has been discussed through figures. It is found that the change in density parameter, adiabatic index of gas and Alfven Mach number of the medium play a significant role on the post detonation front flow variables.

References

[1] R. L. Welsh, "Imploding shocks and detonations," *J Fluid Mech*, p. 61, 1967, Accessed: Feb. 06, 2024. [Online]. Available: https://www.cambridge.org/core/journals/journal-of-fluid-mechanics/article/imploding-shocks-and-detonations/799D6529F787384217498755CA6F6F21

[2] R. I. Nigmatulin, "Converging cylindrical and spherical detonation waves," *Journal of Applied Mathematics and Mechanics*, vol. 31, no. 1, pp. 171–177, Jan. 1967, doi: 10.1016/0021-8928(67)90082-2.

[3] I. Teipel, "Detonation waves in pipes with variable cross-section," *Acta Mech*, vol. 47, no. 3–4, pp. 185–191, Sep. 1983, doi: 10.1007/BF01189207.

[4] J. P. Vishwakarma, G. Nath, and R. K. Srivastava, "Self-similar solution for cylindrical shock waves in a weakly conducting dusty gas," *Ain Shams Engineering Journal*, vol. 9, no. 4. pp. 1717–1730, 2018. doi: 10.1016/j.asej.2016.09.013.

[5] B. G. Verma and J. B. Singh, "Imploding detonation waves," *Def Sci J*, vol. 31, no. jan, pp. 1–6, 1981.

[6] B. G. Verma and J. B. Singh, "Magnetogasdynamic converging spherical detonation waves," *Astrophys Space Sci*, vol. 72, no. 1, pp. 133–142, Sep. 1980, doi: 10.1007/BF00642172.

[7] W. Chester, “The diffraction and reflection of shock waves,” *Quarterly Journal of Mechanics and Applied Mathematics*, vol. 7, no. 1, pp. 57–82, 1954, doi: 10.1093/qjmam/7.1.57.

[8] R. F. Chisnell, “The normal motion of a shock wave through a non-uniform one-dimensional medium,” *Proc R Soc Lond A Math Phys Sci*, vol. 232, no. 1190, pp. 350–370, 1955, doi: 10.1098/rspa.1955.0223.

[9] G. B. Whitham, “On the propagation of shock waves through regions of non-uniform area or flow,” *J Fluid Mech*, vol. 4, no. 4, pp. 337–360, 1958, doi: 10.1017/S0022112058000495.

[10] Christo Ananth, Ho Soon Min, Cheng Siong CHIN, P.Avirajamanjula. "A Brief Outline Of Technical Challenges In Wireless Technology", Rakuten Kobo Inc. Publishing, Toronto, Canada, ISBN: 978-81-910-748-7-1, Volume 6, September 2017, pp: 1-174.

[11] P. K. Gangwar, “Behavior of dusty real gas on adiabatic propagation of cylindrical imploding strong shock waves,” *AIP Conf Proc*, vol. 1953, p. 130024, May 2018, doi: 10.1063/1.5033168.

[12] P. K. Gangwar, “Entropy production on dusty shock propagation in presence of overtaking disturbances,” *AIP Conf Proc*, vol. 2220, p. 120008, 2020, doi: 10.1063/5.0001960.

[13] R. P. Yadav, “Effect of overtaking disturbances on the propagation of strong cylindrical shock in a rotating gas,” *Modelling, Measurement and Control B*, vol. 46, no. 4, pp. 1–11, 1992.

[14] Christo Ananth, Rajini K R Karduri, "Marine Energy Resources: Tapping into the Power of Waves and Tides", International Journal of Advanced Research in Basic Engineering Sciences and Technology (IJARBEST), Volume 7,Issue 1,January 2021,pp 58-66.

[15] H. Li and G. Ben-Dor, “A Modified CCW Theory for Detonation Waves,” *Combust Flame*, vol. 113, no. 1–2, pp. 1–12, Apr. 1998, doi: 10.1016/S0010-2180(97)00136-3.

[16] P. K. Gangwar, “Entropy production on dusty shock propagation in presence of overtaking disturbances,” *AIP Conf Proc*, vol. 2220, no. May, 2020, doi: 10.1063/5.0001960.