A Hybrid Method to Determine Weights of Criteria

Susmita Bandyopadhyay
Department of Business Administration, University of Burdwan, West Bengal, India-713104
bandyopadhyaysusmita2010@gmail.com

Abstract— **This paper proposes a novel hybrid strategy to determine the weights of criteria for Multi-Criteria Decision Analysis (MCDA) techniques. The proposed strategy takes into account the degree of entropy, relationships among criteria, and loss incurred for not giving higher weightages to less weighted criteria. The MCDA techniques which have been hybridized partially are IDOCRIW, CRITIC and CILOS and this has been applied to a case study on ranking alternatives. The ranking of alternatives has been performed through TOPSIS MCDA technique. In order to verify whether the proposed technique is effective, the ranks as obtained with techniques, ODOCRIW, CRITIC, CILOS have been calculated separately. These obtained ranks have been compared with that as obtained out of the proposed hybrid technique. The high level of positive association indicates that the proposed technique provides consistent ranking with the other three rankings.**

Keywords—MCDA Techniques; IDOCRIW; CRITIC; CILOS; Ranking of Alternatives

I. INTRODUCTION

Multi-Criteria Decision Analysis (MCDA) techniques are very popular and well-practiced techniques as evident from the existing literature. These techniques are applied to rank alternatives. The existing literature shows significant number of hybrid techniques along with the hybridization among those techniques. Some of the benchmark MCDA techniques include TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) [1], PROMETHEE (Preference Ranking Organization METHod for Enriched Evaluation) [2], AHP (Analytic Hierarchy Process) [3], ANP (Analytic Network Process) [4], MAUT (Multi-Attribute Utility Theory) [5], ELECTRE-III, (ELimination Et Choix Traduisant la REalité – Elimination of Choice Expressing Reality) [6], VIKOR [7], COPRAS (COmplex PRoportional ASsessment) [8], WASPAS (Weighted Aggregated Sum Product ASsessment) [9], 2012), MACBETH (Measuring Attractiveness by a Categorical Based Evaluation TecHnique) [10], ORESTE [11], EVAMIX [12], ARAS (Additive Ratio ASsessment) [13], MOORA (Multi-Objective Optimization Ratio Analysis) [14], MABAC [15], and MARE [16] and TODIM [17] and so on. All these techniques are used to rank alternatives based on the given criteria. These techniques are also applied to sorting problems. There are several hybrid techniques as well, in which either more than one MCDA techniques have been hybridized or one or more MCDA technique(s) have been hybridized with the other techniques. However, in addition to the techniques for ranking the alternatives, there are MCDA techniques which have been used to determine weights of the criteria. Each of these techniques has separate basis or aspect for determining the weights. For example, IDOCRIW (Integrated Determination of Objective CRIteria Weights) as proposed by Zavadskas et al. [18] is based on the entropy or information content in the criteria; CRITIC (CRiteria Importance Through Intercriteria Correlation) as proposed by Diakoulaki et al. [19] is based on the relationship among the criteria; CILOS [20] is based on the loss incurred due to not assigning higher weightages to the less weighted criteria. However, the existing literature does not show any single MCDA technique which would combine all these three essential characteristics while determining weights of criteria. This paper fills this research gap by combining these three MCDA techniques partially in order to incorporate entropy, relationships among criteria and the aforementioned loss in a single MCDA technique. The proposed technique first determines weights by applying IDOCRIW; and then, determines weights of criteria by applying CRITIC and CILOS partially. The method as applied is described below.

At first, IDOCRIW is applied. Each of the values in the decision matrix is divided by the respective total of the respective criterion in order to get normalized values following expression (1). Then the entropy for each criterion is calculated by expression (2) followed by the calculation of deviation as shown in expression (3). The final weights are determined by normalizing the values as obtained by applying expression (3), as shown in expression (4). Next, CRITIC is applied. At first, values of the decision matrix are normalized by expressions (5) (for benefit type of criteria) and (6) (for cost type of criteria). Then, the correlations among the criteria are determined following expression (7) along with the standard deviation of each criterion as shown in expression (8). Indices for the criteria are then calculated by expression (9). The final weight of each criterion is calculated by expression (10). Next, CILOS is applied. At first, cost type of criteria are converted to benefit type of criteria by dividing the minimum value for each criterion by each of the values as shown in expression (11). Next, find the maximum value for each criterion. Now, form a square matrix $S_{n\times n}$ of the order equal to the number of criteria. The elements of the main diagonal are the maximum values of the criteria and the remaining elements of each row are the remaining elements in the same row of the maximum value in the respective row as the diagonal element. Next, elements of loss matrix are calculated following expression (12). Now, find the mean or median of each row. These are the weights of criteria $w_{CILOS,j}$. The hybridization combines these three weights following expression (13) in order to combine all the characteristics of all these three techniques. For more understanding of these techniques, the work of Alinezhad and Khalili [21] may be consulted.

$$\overline{a_{ij}} = \frac{a_{ij}}{\sum_{i=1}^{m} a_{ij}} \quad (1)$$

$$epy_j = -\frac{1}{\ln m}\sum_{i=1}^{m} \overline{a_{ij}} \times \ln \overline{a_{ij}} \quad (2)$$

$$dev_j = 1 - epy_j \quad (3)$$

$$w_{IDOCRIW,j} = \frac{dev_j}{\sum_{j=1}^{n} dev_j} \quad (4)$$

$$\overline{a_{ij}} = \frac{a_{ij} - a_{\min,j}}{a_{\max,j} - a_{\min,j}} \quad (5)$$

$$\overline{a_{ij}} = \frac{a_{ij} - a_{\max,j}}{a_{\min,j} - a_{\max,j}} \quad (6)$$

$$r_{ij} = \frac{\sum_{i=1}^{m}(a_{ij} - a_{avg,j})(a_{ik} - a_{avg,k})}{\sqrt{\sum_{i=1}^{m}(a_{ij} - a_{avg,j})^2 \sum_{i=1}^{m}(a_{ik} - a_{avg,k})^2}} \quad (7)$$

$$\sigma_j = \sqrt{\frac{1}{n-1}(a_{ij} - a_{avg,j})^2} \quad (8)$$

$$I_j = \sigma_j \sum_{j=1}^{n} r_{ij} \quad (9)$$

$$w_{CRITIC,j} = \frac{I_j}{\sum_{j=1}^{m} I_j} \quad (10)$$

$$\overline{a_{ij}} = \frac{a_{\min,j}}{a_{ij}} \quad (11)$$

$$L_{ij} = \frac{a_{\max,j} - S_{ij}}{a_{\max,j}} \quad (12)$$

$$w_j = w_{IDOCRIW,j} \times w_{CRITIC,j} \times w_{CILOS,j} \quad (13)$$

A case study followed by experimentation results show the effectiveness of the above-mentioned simple strategy

II. Case Study and Experimentation

The case on which the experimentations are accomplished - We all know that “Cigarette smoking is injurious to health” which is a statutory warning as observed in every packet of cigarette. In spite of knowing this fact, the smokers get attracted to cigarettes. What is attractive about cigarette? If taken in small amount, the nicotine content in cigarette causes pleasant feeling and distracts the user from unpleasant feeling. Although nicotine is an additive substance but it is less injurious as compared to tar or carbon monoxide (CO). The tar in cigarette comes of tobacco. Tar is generated when the tobacco in the cigarette burns. The tar mixed with nicotine increases the addition. Carbon monoxide is generated when the tobacco burns incompletely. Carbon monoxide is generated when there is not sufficient oxygen to convert the entire carbon to carbon dioxide which also increases the addition further. The current study is focused on a case study on the ingredients of cigarettes. Different companies in the world have different proportion of ingredients in their cigarette. Based on the given data, a list of brands of cigarettes is provided in Table 1 and the proportions of the ingredients are also provided. The list provides different cigarette brands along with the nicotine content and possible average amount of tar and carbon monoxide which may be generated since the production of these two substances are dependent on the manufacturing design. At first, IDOCRIW, CRITIC and CILOS are applied on Table 1 to get the weights along with the hybridized weight as shown in Table 2.

Table I. LIST OF CIGARETTE BRANDS

Brand	Nicotine	Tar	Carbon Monoxide
Carlton (C1)	0.1	1	1
Merit (C2)	0.3	3	5
Now (C3)	0.2	1	2
Eclipse (C4)	0.2	5	5
Barclay (C5)	0.3	4	4
Quest (C6)	0.3	8	11
Doral (C7)	0.4	4	5
Cimarron (C8)	0.4	4	6
Marlboro (C9)	0.1	1	1
Signature (C10)	0.4	4	5
Camel (C11)	0.4	5	6
Austin (C12)	0.4	5	7
Classic Filter (C13)	0.3	1	1
West White (C14)	0.2	2	3
Virginia Slims (C15)	0.1	1	1
Winston Xsence White Mini (C16)	0.1	1	2

Table II. WEIGHTS OF CRITERIA

	Nicotine	Tar	CO	
IDOCREW	0.3378	0.3313	0.3309	
CRITIC	0.382879	0.306782	0.31034	
CILOS	0.52957	0.262887	0.207543	**TOTAL**
Weights	0.068493	0.026719	0.021313	0.116525
+Normalized Weights	0.587796	0.2293	0.182905	

Now given the weights, at first, the alternatives are ranked by TOPSIS techniques using the hybrid technique followed by IDOCRIW, CRITIC and CILOS respectively. In short, TOPSIS after normalizing and multiplying the normalized values of the decision matrix by the respective weights of the criteria in order to get the weighted normalized decision matrix, finds the best and worst values for each criterion. The best and the worst values for the benefit type of criteria are the maximum and the minimum values whereas for the cost type of criteria, the best and the worst values are the minimum and the maximum values respectively. After this, aggregate Euclidian distances are calculated for each alternative from the best values (d+) and from the worst values (d-). Closeness coefficient (CC) for each alternative is calculated by dividing d- by (d- + d+) and the alternatives are ranked in the descending order of these CC values. For more understanding of TOPSIS, the work of Ishizaka and Nemery (2013) can be consulted. The results (ranks of the alternatives) from applying the proposed hybrid MCDA weightage, IDOCRIW, CRITIC and CILOS are shown in Table 3.

Table III. RANKS AS OBTAINED ON APPLYING IDOCRIW, CRITIC, CILOS AND PROPOSED HYBRID STRATEGY

IDOCRIW	CRITIC	CILOS	HYBRID METHOD
2	2	2	2
9	9	8	8
5	5	5	5
12	12	13	13
8	8	9	9
16	16	16	16
10	10	10	10
13	13	12	12
2	2	2	2
10	10	10	10
14	14	14	14
15	15	15	15
1	1	1	1
7	7	7	7
2	2	2	2
6	6	6	6

In order to verify whether the proposed strategy is effective, Spearman's rank correlation method (expression 14) is applied as shown in Table 4. Table 4 shows high positive rank correlation between the proposed strategy and each of the techniques, IDOCRIW, CRITIC, and CILOS indicating that the proposed strategy is providing very close results as these three other techniques, which established the effectiveness of the proposed strategy.

Table IV. SPEARMAN'S RANK CORRELATION

	IDOCRIW	CRITIC	CILOS
Proposed Strategy	0.99411765	0.99412	1

III. CONCLUSION

This paper has proposed a novel hybrid strategy to find weights of criteria for Multi-Criteria Decision Analysis (MCDA) techniques. The proposed strategy borrows the idea of incorporating information content or entropy from the technique ODOCRIW; relationships among the criteria from CRITIC; and the loss due to not assigning higher weightages to the less weighted criteria, from CILOS. The weights as obtained from these techniques – proposed novel strategy, IDOCRIW, CRITIC, CILOS, have been applied on TOPSIS multi-criteria decision analysis techniques to produce four sets of rankings. Spearman's rank correlation has been applied in order to verify the association among these four sets of rankings. The high positive values of rank correlations indicate very strong association among these four sets of rankings. This establishes that fact that the proposed strategy provides similar ranking as the other three techniques for calculating weights. This, in turn, indicates that instead of applying the techniques, IDOCRIW, CRITIC, CILOS, the proposed strategy can be applied so that all the above-mentioned characteristics can be considered to get a reliable weightages for criteria.

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