

# Design and Analysis of Resonant Harmonic Filter

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**Abstract**—Resonant Harmonic Filter (RHF) are effective passive filter for reducing supply current harmonics by means of creating a low impedance path. Current Harmonics as well as the supply voltage harmonics may deteriorate the efficiency of Resonant Harmonic Filter (RHF). This is because the resonance of the Resonant Harmonic Filter with the distribution system inductance. The performance of a Resonant Harmonic Filter (RHF) is the resultant of the frequency property of the filter and the system and harmonic range of voltage and current. The filter performance can be improved by optimal selection of filter parameter. The frequency property of filter and system can be expressed using the Transmittance Approach.

**Keywords**—Harmonics, Passive Filter, Quality Factor, Transmittance.

## I. INTRODUCTION

Harmonic distortion has a detrimental effect on both distribution system equipment and loads that the system supplies. Because of this, harmonic distortion is a main cause of supply quality deprivation. In addition, current harmonics cause, like reactive current, a deprivation of the power factor. Reduction in the power factor means increased losses during transmission of energy as well as requiring higher ratings of distribution system equipment. For that reason, it is necessary to reduce harmonic currents for the same reasons as for reactive current, as well as to reduce the other harmful effects. If harmonic distortion exceeds some limits, then equipment is needed for its containment. Equipment for harmonic containment should also compensate the reactive current thereby improving both power factor and supply quality [1].

Resonant Harmonic Filter (RHF) are the main device installed in distribution systems for reducing distortion caused by harmonic generating loads. RHF are reactive devices built of resonant LC branches,

connected in parallel to the load. Each branch is tuned to the frequency of a dominating harmonic; therefore, it is a notch filter that provides a low impedance path for the load generated current harmonics to which the branch is tuned. At the same time, the filter provides capacitive reactive power needed to compensate the reactive power of the load[5].

In the few decades before RHF were mainly used for protecting distribution systems against harmonic currents injected by individual harmonic generating loads. Therefore, the resonances were not particularly harmful due to a less dense harmonic spectrum of the load generated current and a relatively distortion free distribution voltage. However, due to rapid propagation of harmonic generating loads in recent years, RHF are installed in distribution systems where the voltage can be substantially distorted. Resonant harmonic filters used in such conditions could be much less effective in reducing supply current distortion. The performance of a RHF is the resultant of the frequency properties of the filter and the system and the harmonic spectrum of the voltage and current. Therefore, the decline in RHF effectiveness may be lessened if the effects of the minor harmonics are taken into account during the filter design process. [3], [6]. The design of the filters should not be performed without specific data on the system and waveform distortion and, therefore, it must be done on a step-by-step basis. Once the system and range are identified, harmonic amplification and attenuation by RHF can be explained in terms of filter transmittances which describe the frequency properties of the filter and the system. The transmittances approaches allow the definition of distortion coefficients and the filter performance measures. In addition, by enlightening specific causes for a filter's decline in performance, the transmittance approaches facilitate the prediction of the performance under various conditions [6]. By using this transmittances approach, methods of modifying RHF filter characteristics will be determined and evaluated.

II. ANALYSIS OF PASSIVE FILTER

The harmonic filters shunt elements that are used in power systems for decreasing voltage distortion and for power factor correction. Nonlinear elements such as power electronic converters generate harmonic currents or harmonic voltages, which are injected into power system. The resulting distorted currents flowing through system impedance produce [2] harmonic voltage distortion. Harmonic filters reduce distortion by diverting harmonic currents in low impedance paths. Harmonic filters are designed to be capacitive at fundamental frequency, so that they are also used for producing reactive power required by converters and for power factor correction [8].

The passive harmonics filters are composed of passive elements: resistor (*R*), inductor (*L*) and capacitor (*C*). The common types of passive harmonic filter include single-tuned and double-tuned filters, second-order, third-order and C-type damped filters. The double-tuned filter is equivalent to two single-tuned filters connected in parallel with each other, so that only single-tuned filter and other three types of damped filters are shown in figure.1.



Figure.1 Types of Filter

A. Design Of Passive Filter

The simplest filter type is the single-tuned filter. The quality factor of the reactance at the tuning frequency is  $Q_f$  [3], [4].

$$Q_f = \frac{Q_C}{P_L} \times \frac{h}{h^2 - 1} = \frac{R \times X_L}{R} = \frac{X_C}{h \times R} \quad (1)$$

The inductor reactance at fundamental frequency =  $\omega L$ .

$$X_C = \frac{V_L^2}{Q_C} \times \frac{h^2}{h^2 - 1} \quad (2)$$

The capacitor reactance at fundamental frequency =  $1/\omega C$ .

$$X_L = \frac{X_C}{h^2} \quad (3)$$

Tuned harmonic order,

$$h = \sqrt{\frac{X_C}{X_L}} \quad (4)$$

The inductance and capacitance of the branch which the harmonic order to be tuned with respective to load.

$$L = \frac{X_L}{2 \times \pi \times f_0} \quad (5)$$

$$C = \frac{1}{2 \times \pi \times f_0 \times X_C} \quad (6)$$

- Where,
- $Q_C$ -Reactive capacitive power.
- $V_L$ -Load voltage.
- $R$ -Resistance.
- $L$ -Inductor.
- $C$ -Capacitor.
- $f_0$ -Fundamental frequency.
- $h$ -Order of Harmonic.

Case Study: 1:

In this case study considering a 3-phase, 50Hz system with the load voltage of 315kV and reactive power of 49MVAR. Based on the calculation procedure, design of a single tuned filter for different order of harmonics like 3<sup>rd</sup>, 5<sup>th</sup>, 7<sup>th</sup> and 9<sup>th</sup> can be performed. Table.1, shows the element values for different harmonic order based on quality factor. Based on these element values a single tuned filter network is designed and shown in figure.2. And the corresponding Impedance Vs Frequency graph is shown in figure.3. Which will clearly indicates the low impedance path for the corresponding order of harmonics.

TABLE.1 IMPEDANCE OF SINGLE TUNED FILTER

Harmonic Order	Quality Factor	Resistor (ohm)	Inductor (H)	Capacitor (µf)
3 <sup>rd</sup>	54.0441	14.0510	0.8057	1.3972
5 <sup>th</sup>	30.0245	14.0510	0.2686	1.5090
7 <sup>th</sup>	21.0172	14.0510	0.1343	1.5398
9 <sup>th</sup>	16.2132	14.0510	0.0806	1.5525

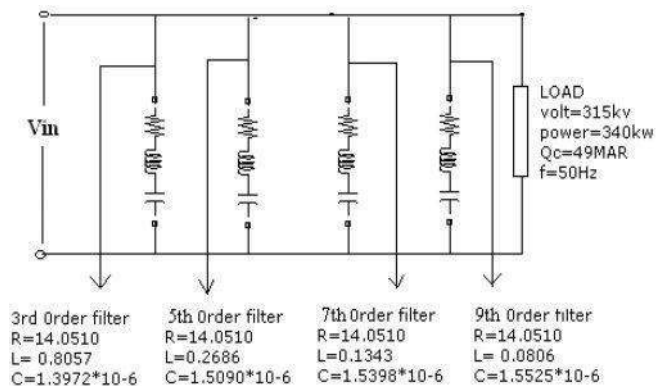


Figure.2 Single Tuned Filter Network

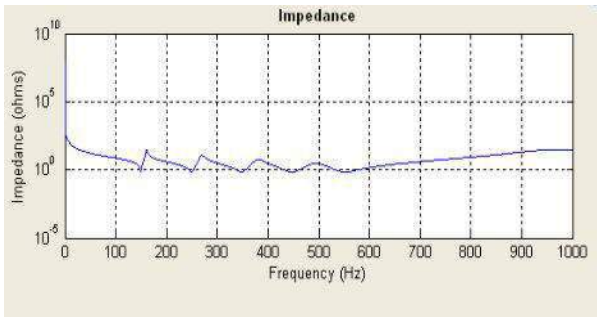


Figure.3 Impedance Vs Frequency Graph

### III. FUNDAMENTALS OF RESONANT HARMONIC FILTER

Conventional harmonic filters are reactive devices built of resonant LC-branches, connected in parallel to the load. Each branch is tuned to a specific harmonic frequency or in its vicinity, and therefore each branch provides a low impedance path for a single load generated harmonic current. This allows the harmonic current to bypass the supply, thus the supply current and voltage are not affected. At the same time, the filter provides reactive power compensation for the load.

In recent years Resonant Harmonic Filter are installed in distribution systems where there is a large number of power electronic equipment that generate current harmonics, and consequently, the distribution voltage can be substantially distorted [5]. Moreover, the load current may have a dense harmonic spectrum that means, harmonics other than those to which the filter is tuned may exist in the load current and have a substantial value. Such current harmonics and any harmonics in the distribution system voltage are referred to as minor harmonics.

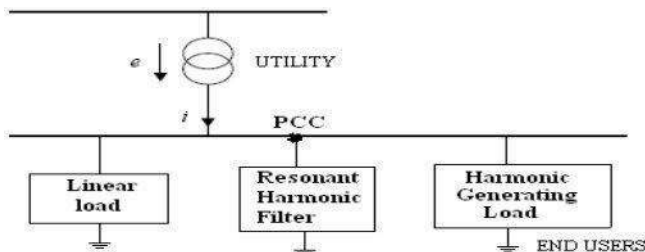


Figure.4 One Line diagram of a system with a RHF Branch

The method of design of RHF as well as the reasons for the decline in effectiveness when minor harmonics are present. The reasons for decline in effectiveness are best described using the filter transmittances which describe the frequency properties of the filter and the system. The transmittances also allow the definition of distortion coefficients and filter performance measures.

### IV. TRADITIONAL DESIGN OF RESONANT HARMONIC FILTER

The parameters of individual branches of the Resonant Harmonic Filter are calculated based on the value of the reactive power compensated by a branch and the chosen resonant frequency of the branch. This frequency, to distinguish it from the frequency of the filter resonance with the distribution system, will be referred to as a tuning frequency. Each branch of a RHF has capacitive impedance at the fundamental frequency [6]. Thus, each branch of a RHF compensates a portion of the load reactive power  $Q_1$  at the fundamental frequency. If a filter has  $K$  branches then the reactive power compensated by one branch, denoted  $Q_{1k}$ , can be expressed as

$$Q_{1k} = d_k \times Q_1 \tag{7}$$

The coefficient  $d_k$  is the reactive power allocation coefficient. It has a value between 0 and 1 corresponding to the percentage of reactive power compensated by the branch, and it may be chosen at the designer's caution.

$$Q_{tot} = Q_1 \sum_{k=1}^K d_k \tag{8}$$

The LC branch which has a high quality factor, resistance in the branch can be neglected and the branch susceptance can be expressed as

$$B_k = \text{Im} \left( \frac{1}{j\omega_1 L_k + \frac{1}{j\omega_1 C_k}} \right) = \frac{\omega_1 C_k}{1 - \omega_1^2 L_k C_k} \tag{9}$$

The branch tuned to the frequency of  $\zeta \omega_1$  to provide a low impedance path for a harmonic of order  $n$ , then

$$L_k C_k = \frac{1}{\zeta_k^2 \omega_1^2} \tag{10}$$

The capacitance and inductance of the branch are determined by using the formula,

$$C_k = \frac{d_k Q_1 \left( 1 - \frac{1}{\zeta_k^2} \right)}{\omega_1 U^2} \tag{11}$$

$$L_k = \frac{1}{d_k Q_1 \omega_1 (\zeta_k^2 - 1)} \tag{12}$$

Where,

$\zeta_k \rightarrow$  order of harmonic.

$U \rightarrow$  supply voltage.

$\omega_1 \rightarrow$  Fundamental frequency.

$Q_1 \rightarrow$  Reactive power of the load.

$d_k \rightarrow$  Reactive power allocation co-efficient.

A. Location of Resonant Frequency  $\omega_r$ :

In order to adjust filter parameters for the purpose of avoiding resonance at harmonic frequencies, the relation between reactive power allocation and resonant frequency locations is needed. The quality factor of filter inductors is usually very high for RHF and supply and load inductance dominate the supply and load impedance at harmonic frequencies. Therefore, to find the resonant frequency, considering a reactive equivalent circuit.

The equivalent network as seen by the supply for such a circuit having a filter with  $K$  branches is shown below in Figure 5.

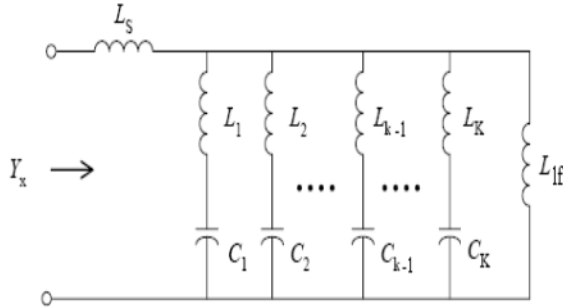


Figure.5 Equivalent one-port network

The lumped impedance of the filter branches and the load equivalent inductance  $L_{lf}$  are connected in series with the equivalent supply inductance  $L_s$  [5].

$$\omega_r^2 = \frac{1}{2} \left\{ \frac{y_1}{y_2} \pm \sqrt{\left(\frac{y_1}{y_2}\right)^2 - 4 \frac{y_0}{y_2}} \right\} \quad (13)$$

Where,

$$y_2 = L_{lf} L_s \left( \left( \frac{1}{L_{lf}} + \frac{1}{L_s} \right) + a_1 z_1^2 + a_2 z_2^2 \right) \quad (14)$$

$$y_1 = L_{lf} L_s \left( \left( \frac{1}{L_{lf}} + \frac{1}{L_s} \right) (z_1^2 + z_2^2) + (z_1 z_2)^2 (a_1 + a_2) \right) \quad (15)$$

$$y_0 = L_{lf} L_s \left( \left( \frac{1}{L_{lf}} + \frac{1}{L_s} \right) (z_1 z_2)^2 \right) \quad (16)$$

**B. Transmittance Approach :**

The performance of a RHF is the resultant of the frequency properties of the filter and the system and the harmonic range of the voltage and current. The frequency properties of the filter and the system can be expressed using the transmittance approach [6].

A simplified model of a system with an inductive reactance in the supply impedance is shown in figure.

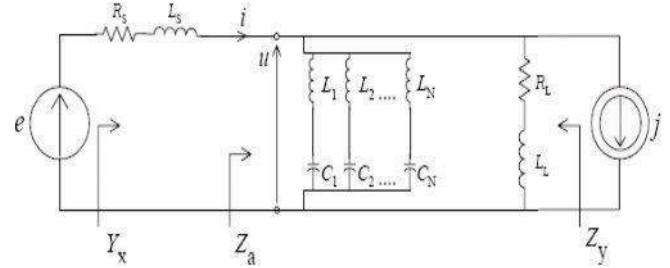


Figure.6 Equivalent Circuit of Resonant Harmonic Filter

The transmittances,  $A(j\omega)$ ,  $B(j\omega)$ ,  $Y_x(j\omega)$ , and  $Z_y(j\omega)$ , are defined as

$$A(j\omega) = \frac{U(j\omega)}{E(j\omega)} = \frac{z_a(j\omega)}{z_f(j\omega) + z_a(j\omega)} \quad (17)$$

$$B(j\omega) = \frac{I(j\omega)}{J(j\omega)} = \frac{z_a(j\omega)}{z_f(j\omega) + z_a(j\omega)} \quad (18)$$

$$Y_x(j\omega) = \frac{I(j\omega)}{E(j\omega)} = \frac{1}{z_f(j\omega) + z_a(j\omega)} \quad (19)$$

$$Z_y(j\omega) = \frac{I(j\omega)}{E(j\omega)} = \frac{z_f(j\omega) z_a(j\omega)}{z_f(j\omega) + z_a(j\omega)} \quad (20)$$

Where,

$$z_a(j\omega) = \frac{z_f(j\omega) z_l(j\omega)}{z_f(j\omega) + z_l(j\omega)} \quad (21)$$

$A(j\omega) \rightarrow$  Ratio of spectra of bus voltage  $U$  & distribution Voltage  $E$ .

$B(j\omega) \rightarrow$  Ratio of spectra of supply current  $I$  & load Current  $J$ .

$Y_x(j\omega) \rightarrow$  Admittance for harmonic frequency.

$Z_y(j\omega) \rightarrow$  Impedance for harmonic frequency.

$Z_s(j\omega) \rightarrow$  Source impedance.

$Z_f(j\omega) \rightarrow$  Filter impedance.

$Z_L(j\omega) \rightarrow$  Load impedance.

$Z_a(j\omega) \rightarrow$  Impedance of Filter and Load.

The source resistance  $R_s$  is determined by the formula

$$R_s = \frac{U^2}{S_{sc} \sqrt{1 + \xi_s^2}}, \text{ where } \xi_s = \frac{X_s}{R_s} \quad (22)$$

The equivalent load susceptance  $B_L$  is a sum of the filter and load susceptance. The load susceptance at frequency  $\omega$  is equal to

$$B_L = -\frac{1}{R_L} \frac{r \tan \phi}{1 + r^2 \tan^2 \phi} \quad (23)$$

Where,

$$R_L = \frac{U^2 \lambda^2}{P} \quad (24)$$

$$\phi = \cos^{-1} \lambda \quad (25)$$

$$\gamma = \frac{\omega_r}{\omega_1}, \tag{26}$$

The load susceptance can be approximated by

$$B_L = \frac{1}{R_L \gamma \tan \varphi} ; X_L = \frac{1}{B_L} \tag{27}$$

Where,

$\lambda$  – Power factor.

P – Reactive power.

The filter susceptance for each k<sup>th</sup> branch,

$$B_{K'} = \frac{\omega_r C_{K'}}{1 - \omega_1^2 L_{K'} C_{K'}} ; X_{K'} = \frac{1}{B_{K'}} \tag{28}$$

The filter susceptance is approximately equal to

$$B_f = \sum_{k=1}^K B_{K'} ; X_f = \frac{1}{B_f} \tag{29}$$

The load conductance at resonant frequency is equal to

$$G_L = \frac{P}{\gamma^2 (1 - \lambda^2) U^2} ; R_L = \frac{1}{G_L} \tag{30}$$

The filter conductance for each k<sup>th</sup> branch,

$$G_k = \frac{\frac{n_k \omega_1}{\omega_1 n_k} \frac{1}{U^2 q_k}}{\left(\frac{n_k \omega_1}{\omega_1 n_k}\right)^2 U^2 q_k} ; R_k = \frac{1}{G_k} \tag{31}$$

The filter conductance is approximately equal to

$$G_f = \sum_{k=1}^K G_k ; R_f = \frac{1}{G_f} \tag{32}$$

The total load impedance,

$$Z_L = R_L + jX_L \tag{33}$$

C. Implementation of Resonant Harmonic Filter :

Case Study: 2:

A reactive load at the supply voltage assumed to be  $U = 1$  pu and the apparent power  $S = 1$  pu is supplied from a bus with the short circuit power  $S_{sc} = 40$  pu,  $\omega_1 = 1$  rad/sec, the PF equal to  $\lambda = 0.707$ , the reactance to resistance ratio at the bus is assumed to be  $\xi_s = X_s/R_s = 5$ . The filter has two branches, tuned to the 5<sup>th</sup> and 7<sup>th</sup> order harmonics, thus  $\Omega_1 = 5$  rad/sec,  $\Omega_2 = 7$  rad/sec. The damping coefficients are calculated for two resonant frequencies that coincide with the 4<sup>th</sup> and the 6<sup>th</sup> order harmonics and for two different Quality factors, the same for each branch, namely,  $Q = 100$  and  $Q = 30$ .

The conductance and susceptance of the load and the filter branches for resonant frequencies are compiled and tabulated. The results compiled show that for a common level of the filter q-factor, the load resistance has much lower contribution to the resonance damping than the filter resistance[5]&[6].By using the Conductance and susceptance of the load and the filter branch values from table.2 the Transmittance Approach is examined, tabulated and graph has been plotted .

TABLE .2 CONDUCTANCE AND SUSCEPTANCE OF THE LOAD AND THE FILTER BRANCHES FOR RESONANT FREQUENCIES.

Susuptance and conductance	rad/sec	4 <sup>th</sup> order resonant Freq		6 <sup>th</sup> order resonant Freq	
G <sub>L</sub>	pu	0.125		0.055	
B <sub>L</sub>	pu	-0.50		-0.33	
B <sub>1</sub>	pu	5.296		-6.63	
B <sub>2</sub>	pu	2.909		11.09	
B <sub>F</sub> (or) B <sub>a</sub>	pu	8.205		4.46	
q-factor	-	100	30	100	30
G <sub>1</sub>	pu	0.118	0.395	0.179	0.595
G <sub>2</sub>	pu	0.024	0.08	0.35	1.193
G <sub>F</sub> (or)G <sub>a</sub>	pu	0.143	0.477	0.529	1.788

TABLE.3 TRANSMITTANCE APPROACH FOR VARIOUS QUALITY FACTOR FOR RESONATE FREQUENCY ( $\omega_r = 4$  &  $\omega_r = 6$ )

Transmittance Approach	Q-factor	$\omega_r = 4$	$\omega_r = 6$
A(j $\omega_r$ / $\omega_1$ ) = B(j $\omega_r$ / $\omega_1$ )	$\infty$	356.28	63.413
	100	71.2567	12.68
	30	21.377	3.8
Yx (j $\omega$ , $\omega_1$ )	$\infty$	148.6	98.215
	100	123.634	41.49
	30	82.91	15.455
Zy(j $\omega_r$ / $\omega_1$ )	$\infty$	1.76	3.186
	100	1.47	1.345
	30	0.9858	0.5014

Based on the various values of Quality factor and  $\omega_r$  with respect to frequency, the magnitude Vs frequency graph is plotted and shown in figure 7-12.

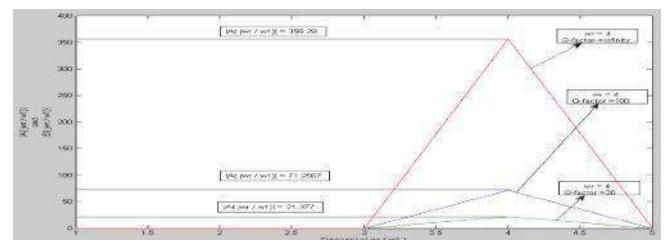


Figure.7 Magnitude of the A(j ω) and B(j ω) transmittances for  $\omega_r=4$ , Quality factor=  $\infty$ , 100 & 30.



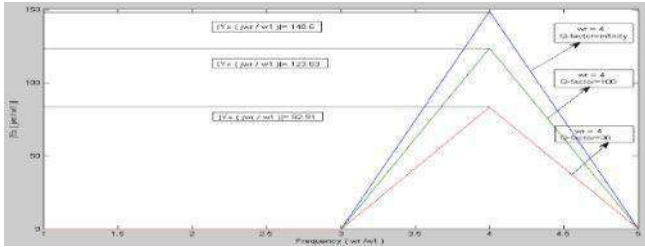


Figure.8 Magnitude of the  $Y_x(j \omega)$  transmittances for  $\omega_r=4$ , Quality factor= $\infty$ , 100 & 30.

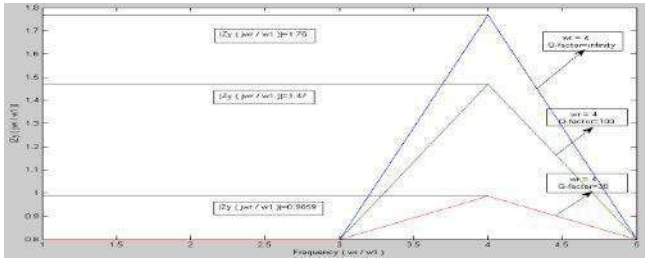


Figure.9 Magnitude of the  $Z_y(j \omega)$  transmittances for  $\omega_r=4$ , Quality factor= $\infty$ , 100 & 30.

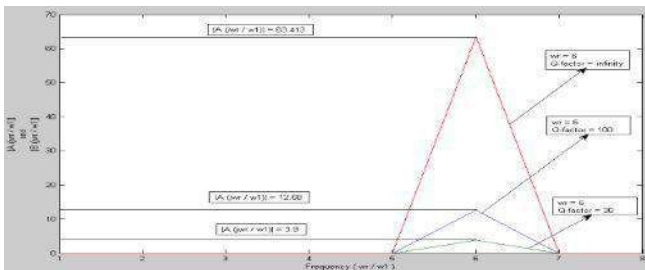


Figure.10 Magnitude of the  $A(j \omega)$  and  $B(j \omega)$  transmittances for  $\omega_r=6$ , Quality factor= $\infty$ , 100 & 30.

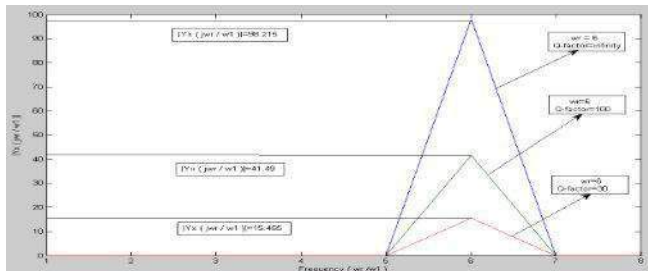


Figure.11 Magnitude of the  $Y_x(j \omega)$  transmittances for  $\omega_r=6$ , Quality factor= $\infty$ , 100 & 30.

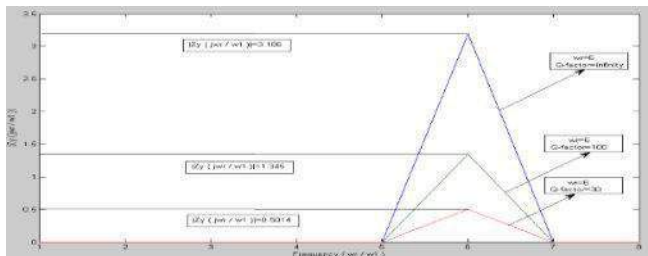


Figure.12 Magnitude of the  $Z_y(j \omega)$  transmittances for  $\omega_r=6$ , Quality factor= $\infty$ , 100 & 30.

## V. CONCLUSION

The passive single tuned filter is designed and it creates a low impedance path for the order of harmonics like 3<sup>rd</sup>, 5<sup>th</sup>, 7<sup>th</sup> and 9<sup>th</sup> for the fixed load value. Due to load variation induced harmonics effects also varying. This could be overcome by Resonant Harmonic Filter. Resonant Harmonic Filter (RHF) is designed for the maximum value in 1 p.u and analyzed by Transmittance Approach method for various values of  $\omega_r$  and Quality factor. Optimization techniques have been recommended for tuning of Resonant Harmonic Filter.

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