PID Control of Twin Rotor MIMO system

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Abstract— **Control design of a multiple input system such as helicopter is very challenging task. This is because of the high non-linearity, significant cross-coupling and inaccessibility of some of its states and outputs for measurements. MIMO control design has been explored in the laboratory with the help of Twin Rotor MIMO system (TRMS). A Twin rotor MIMO system consists of two rotors called main rotor and tail rotor. These two rotors are situated on a beam, supported by a counter balance. Two degrees of freedom (DOF) dynamic model of the TRMS involving pitch and yaw motion was obtained for controller design. PID controllers were designed for TRMS by Zeigler Nichol's tuning method. The root locus of the closed loop system ensured that even if the gain K value was increased, the root locus remained in the stable region. Thus, the tracking error of the TRMS model was reduced to less than 1% with the help of properly designed PID controllers. The study of Eigen Structure Assignment was conducted and is to be implemented to the TRMS model which is considered. A comparative study of the methods is to be carried out, which is included as future work.**

Index Terms—, **PID controller, parameter variations, uncertainties, Twin Rotor MIMO systems(TRMS).**

I. INTRODUCTION

TWIN ROTOR MIMO system consists of two rotors called main rotor and tail rotor. These two rotors are situated on a beam together with a counterbalance as shown in Fig.1.1.The main rotor is free to rotate in the horizontal plane in which angle of rotation is called pitch and the tail rotor is free to rotate in the vertical plane in which angle of rotation is called yaw. The twin rotor system is equivalent to a helicopter in certain aspects. This system has two degree of freedom (2-DOF) on pitch and yaw angles. Twin rotor system attracts control engineers and researchers because it has the presence of uncertainty, complexity, parameter variations, and external disturbances. In recent years, a lot of approaches have been devised for the control of twin rotor system. An adaptive second order sliding mode (SOSM) controller is proposed to control a laboratory helicopter twin rotor MIMO system (TRMS) [1]. The decoupling control of a TRMS is studied and proposed to apply robust deadbeat control technique to this nonlinear system [2].

A decoupled compensator has been designed for a physical TRMS system [3]. For control of TRMS with 2-DOF, a modified real-value-type genetic algorithm (RGA) for tuning the parameters of the proportional integral derivative (PID) controller is used [4]. A fuzzy sliding and fuzzy integral sliding controller (FSFISC) is designed to position the yaw and pitch angles of the TRMS system [5]. A fuzzy PID technique with RGA to control a TRMS has been discussed [6]. A TRMS is decoupled and fuzzy Takagi-Sugeno (T-S) model of TRMS is obtained [7]. On the basis of T-S model, a parallel distributed fuzzy linear quadratic regulator (LQR) controller is designed to control the position of the pitch and yaw angles. Implementation of an adaptive dynamic nonlinear model inversion control law for a TRMS using artificial neural networks (ANN) has been discussed [8].

An observer for nonlinear TRMS has been implemented in [9]. The nonlinearities which are unknown are estimated by Chebyshev neural network whose weights are adaptively adjusted. A quasi-linear parameter varying (quasi-LPV) modeling, identification and control of a TRMS is presented [10]. A twin rotor system is modeled using an ant colony optimization (ACO) technique [11]. A robust adaptive tracking control scheme is developed using back-stepping technique and the dynamic surface control method for a nonlinear MIMO system [12].

This paper is aimed at designing a controller for compensating the parameter variations and external disturbances existing in both pitch and yaw motion of the TRMS. The trajectory tracking controller proposed in this paper uses a PID controller, which is designed by Zeigler Nichol's tuning method. There are two types of tuning methods, out of which we have selected Zeigler Nichol's second method of tuning the PID controllers due to the presence of open loop poles at the origin for the TRMS system which have been modeled by considering 7 states. Also a state feedback control law is to be implemented to the system using Eigen structure assignment [13] so that the desired poles will be the eigen values of the closed loop system. Now these results are to be compared with the Relative Gain Array approach[14]. Here the stability of the system is ensured by plotting the root locus of the closed loop system. By plotting this, we can see the root locus lies on the left half of s-plane. A comparison between the non-linear model and the linear one has been done and the responses were compared both for the open loop and closed loop system.

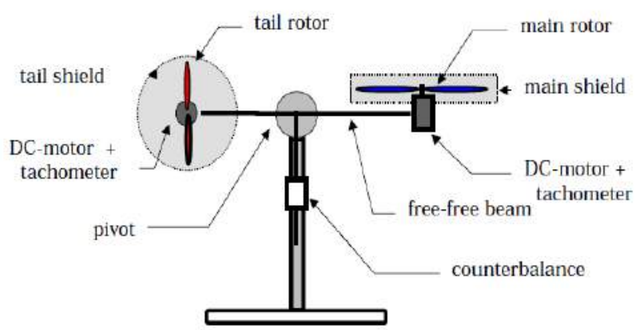


Fig. 1.1 Aero-dynamical model of TRMS

The simulations were conducted in Matlab by providing step input of unity as the inputs to two rotors. The responses of the pitch and yaw motion of the two rotors were compared for both linear as well as for the non-linear system.

The outline of this paper is as follows. Section II is the model description of TRMS; Section III involves the design and simulation of PID controllers for both the rotors. Concluding remarks are given in Section IV.

II. Model Description

TRMS consists of two propellers which are perpendicular to each other and joined by a beam pivoted on its base. The system can rotate freely in both vertical and horizontal direction. Both propellers are driven by DC motor and by changing the voltage supplied to beam, rotational speed of propellers can be controlled. For balancing the beam in steady state, counterweight is connected to the system. Both propellers are shielded so that the environmental effects can be minimized. The complete unit is attached to the tower which ensures safe helicopter control experiments. The electrical unit is placed under the tower which is responsible for communication between TRMS and PC. The electrical unit is responsible for transfer of measured signal by sensors to PC and transfer of control signal via I/O card. Main rotor is responsible for controlling the flight of TRMS in vertical direction and Tail rotor is responsible for controlling the flight of TRMS in horizontal direction.

The mathematical model derived from phenomenological model shown in Fig 2.1 is nonlinear in nature that means at least one of the states (rotor current or position) is an argument of non-linear function. In order to design the controller for controlling the flight of TRMS, the mathematical model should be linearized. According to model represented in Fig1.2, the non-linear mathematical model of TRMS are formed and are shown below:

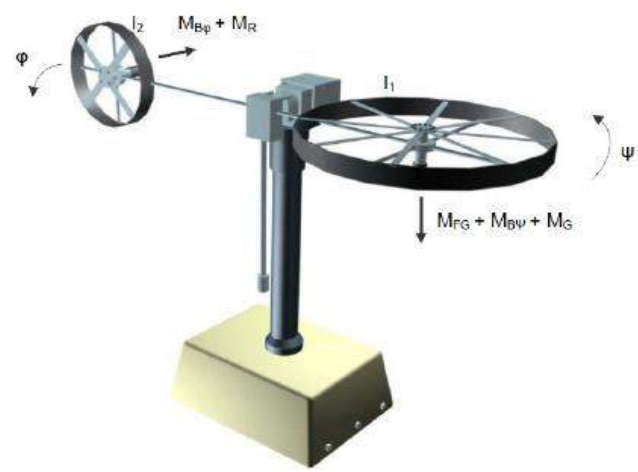


Fig. 2.1 Mechanical-Electrical model of TRMS

Mathematical equation in vertical plane is given by:

$$
\begin{aligned}
I_1 \ddot{\psi} &= M_1 - M_{FG} - M_{B\psi} - M_G \\
M_1 &= a_1 \tau_1^2 + b_1 \tau_1 \\
M_{FG} &= M_g \sin\psi \\
M_{B\psi} &= B_{1\psi} \dot{\psi} + B_{2\psi} sign(\dot{\psi}) \\
M_G &= K_{gy} M_1 \dot{\psi} \cos(\psi)
\end{aligned}
\tag{1}
$$

Where M_1 is the non-linear static characteristics, M_G is the gryoscopic momentum and M_{FG} is the gravity momentum.

The motor and the electrical control circuit is approximated as a first order transfer function, thus the main rotor momentum in Laplace domain is described as-

$$
\tau_1 = \frac{k_1}{T_{11}s + T_{10}} u_1 \tag{2}
$$

Mathematical equation in horizontal plane is given as-

$$
\begin{aligned}
I_2 \ddot{\phi} &= M_2 - M_{B\phi} - M_R \\
M_2 &= a_2 \tau_2^2 + b_2 \tau_2 \\
M_{B\phi} &= B_{1\phi} \dot{\phi} + B_{2\phi} sign(\dot{\phi}) \\
M_R &= \frac{k_c (T_0 s + 1)}{(T_p s + 1)} \tau_1
\end{aligned}
\tag{3}
$$

Where M_R is the cross reaction momentum, and the tail rotor momentum in Laplace domain is given as-

$$\tau_2 = \frac{k_2}{T_{21}s+T_{20}} u_2 \qquad (4)$$

TABLE I
NOMINAL PARAMETERS OF A TRMS

PARAMETER	VALUE
I_1 – momemt of inertia of vertical rotor	$6.8.10^{-2}\ kg.m^2$
I_2 – momemt of inertia of horizontall rotor	$2.10^{-2}\ kg.m^2$
a_1 – static characteristic parameter	0.0135
b_1 – static characteristic parameter	0.0924
a_2 – static characteristic parameter	0.02
b_2 – static characteristic parameter	0.09
M_g – gravity momentum	0.32N.m
$B_{1\psi}$ – friction momentum function parameter	$6.10^{-3}\ N.m.s/rad$
$B_{2\psi}$ – friction momentum function parameter	$1.10^{-3}\ N.m.s^2/rad$
$B_{1\phi}$ – friction momentum function parameter	$1.10^{-1}\ N.m.s/rad$
$B_{2\phi}$ – friction momentum function parameter	$1.10^{-2}\ N.m.s^2/rad$
K_{gy} – gryoscopic momentum parameter	$0.05\ s/rad$
k_1 – motor 1 gain	1.1
k_2 – motor 2 gain	0.8
T_{11} – motor 1 deno parameter	1.1
T_{10} – motor 1 deno parameter	1
T_{21} – motor 2 deno parameter	1
T_{20} – motor 2 deno parameter	1
T_p – cross reaction momentum parameter	2
T_0 – cross reaction momentum parameter	3.5
k_c – cross reaction momentum gain	-0.2

The table I gives the approximate values of parameters.

A. *State-Space Representation*

By using the dynamical equations, the state space model of the linearised plant is given by

$$\dot{x} = Ax + Bu \qquad (5)$$
$$y = Cx + Du$$

The State vector is:

$$x = [\psi\ \ \dot{\psi}\ \ \phi\ \ \dot{\phi}\ \ M_R\ \ \tau_1\ \ \tau_2]^T \qquad (6)$$

The input vector is:

$$u = [u_1\ \ u_2]^T \qquad (7)$$

The output vector is:

$$y = [\psi\ \ \phi]^T \qquad (8)$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{-M_g}{I_1} & \frac{-B_{1\psi}}{I_1} & 0 & 0 & 0 & \frac{b_1}{I_1} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{-B_{1\phi}}{I_2} & \frac{-1}{I_2} & 0 & \frac{b_2}{I_2} \\ 0 & 0 & 0 & 0 & \frac{-1}{T_p} & \frac{k_c}{T_p}\left(\frac{-T_{10}}{T_{11}}T_0+1\right) & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{-T_{10}}{T_{11}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{-T_{20}}{T_{11}} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{k_1 k_C T_0}{T_{11} T_P} & 0 \\ \frac{k_1}{T_{11}} & 0 \\ 0 & \frac{k_2}{T_{21}} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

By substituting the values given in the table for the A,B,C and D matrices, we obtain the values as given below:

State Transition Matrix as:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -4.7059 & -0.088 & 0 & 0 & 0 & 1.359 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -5 & -50 & 0 & 4.5 \\ 0 & 0 & 0 & 0 & -0.5 & 0.22 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.909 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

Input matrix as :

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -0.35 & 0 \\ 1 & 0 \\ 0 & 0.8 \end{bmatrix}$$

Output matrix as:

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

B. Transfer Function Model

The transfer function model of the TRMS is obtained by converting the state-space model into equivalent transfer function by using Matlab commands. Thus we get two transfer functions representing the pitch and yaw rotors as given below:

Pitch rotor transfer function:

$$G_P = \frac{1.359}{s^3 + 0.997s^2 + 4.786s + 4.278} \quad (9)$$

Yaw rotor transfer function:

$$G_Y = \frac{3.6}{s^3 + 6s^2 + 5s} \quad (10)$$

Four transfer functions are obtained, out of which two of them are formed due to the significant cross coupling between the two rotors. We are not much bothered about the remaining transfer functions, since we decoupling the MIMO system into two SISO models, and designing the controllers separately.

III. DESIGN OF PID CONTROLLERS

In this section, we are dealing with the design of PID controllers by means of Zeigler Nichol's tuning method. Before going deep into the controller design part we shall first discuss about the technique using root locus approach. In pole placement technique, we are placing the poles of the system in such a way that the root locus of the closed loop system will pass through the desired pole location, that we have selected by meeting some of the performance specifications. Here we are selecting the performance specifications as settling time equal to 4seconds and percentage overshoot equal to 25%. Thus we get the natural frequency of oscillation $\omega_n = 2.4471\ rad/\sec$ and damping coefficient $\varepsilon = 0.403$.

By plotting the root locus of the open loop system, we have found that system is stable for only for some range of K. Since the open loop plant has an integrator we use Z-N second method. Here initially we set integral time and derivative time as $T_i = \infty$ *and* $T_d = 0$ respectively. The K_p value is increased from 0 to a point at which sustained oscillations are obtained. The time period for which is called the critical time period represented by P_{cr}.

For the sake of convenience we shall discuss how the method is implemented for the yaw rotor.

A. Zeigler Nichol's Tuning Method

Let us consider the root locus of the open loop transfer function of yaw rotor.

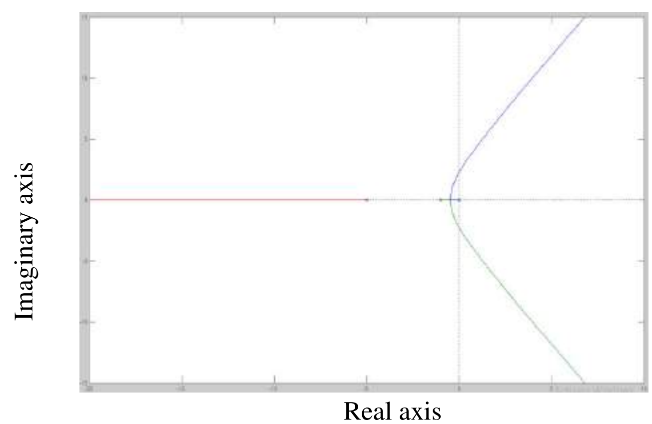


Fig. 3.1 Root locus of the open loop transfer function of Yaw rotor.

The open loop plant is stable for only some value of K, so in order to make the plant globally stable, we design PID controller values my meeting the performance specifications mentioned earlier. Here the second method of Z-N tuning method is used for designing PID controller due to the presence of a pole at the origin.

TABLE II
PARAMETERS USED IN ZEIGLER-NICHOL'S SECOND METHOD

TYPE OF CONTROLLER	K_p	T_i	T_d
P	$0.5K_{cr}$	∞	0
PI	$0.45K_{cr}$	$1/(1.2P_{cr})$	0
PID	$0.6K_{cr}$	$0.5P_{cr}$	$0.125P_{cr}$

Here K_p value is increased from 0 to a particular value where sustained oscillations are obtained. For the yaw rotor the $K_p = 24.43$ is the critical gain.
So we assume $K_{cr} = K_p$ and the time period of oscillation is called critical time period represented by P_{cr} .

For the ideal case, PID controller combination is taken as cascaded form, ie,

$$Gc = K_p(1 + T_d s + \frac{1}{T_i s}) \qquad (11)$$

By substituting the value of K_p in the above equation and equating to zero we get T_i value as equal to 1.4. and T_d = 0.0625.
In the same way PID controller is designed for the Pitch rotor also.

TABLE III
VALUES OF P, I & D FOR THE ROTORS

Rotor type	K_p	K_i	K_d
Pitch	2.74	53.44	11.37
Yaw	24.43	0.72	0.0625

Here K_i is taken as reciprocal of T_i and $K_d = T_d$

B. Block diagram representation

After successful modeling and design of PID controllers for the TRMS, we need to compare the results of both linear as well as the non linear system, whether they are showing similar response towards a step input applied to them. For that we use the following block diagram.

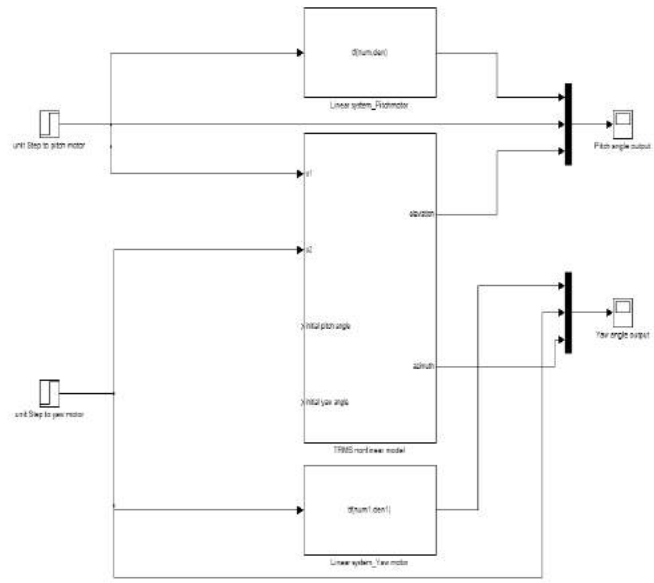


Fig. 3.2 Combined block diagram for the linear and non-linear system without controller.

Combined linear and non-linear block diagram representation of TRMS is shown in Fig 3.4. This is the open loop block diagram , and the responses shown that both linear and the non- linear systems are having similar open loop characteristics. Now we need to confirm the similarity in the closed loop case ,by satisfying the performance specifications.

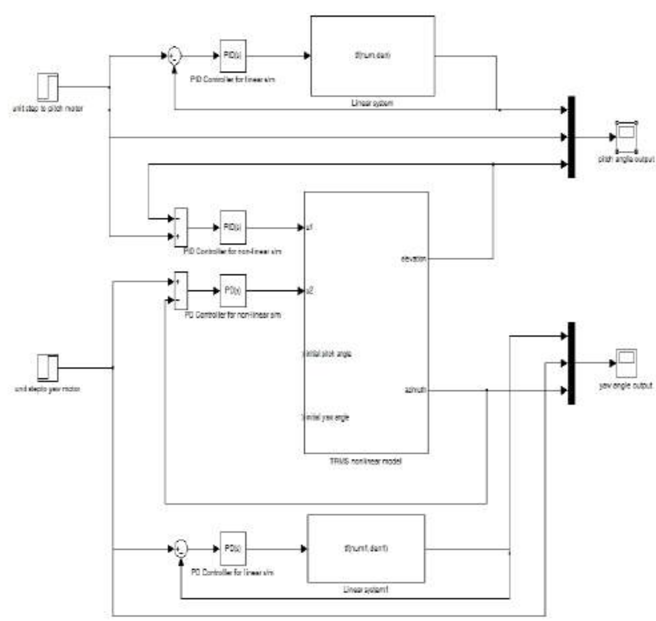


Fig. 3.3 Combined block diagram for the linear and non-linear system with PID controller.

The comparison of closed loop system was also done successfully ie., the closed loop systems of the pitch and yaw rotor for both linear and non-linear systems are showing similar responses satisfying the defined performance specifications.

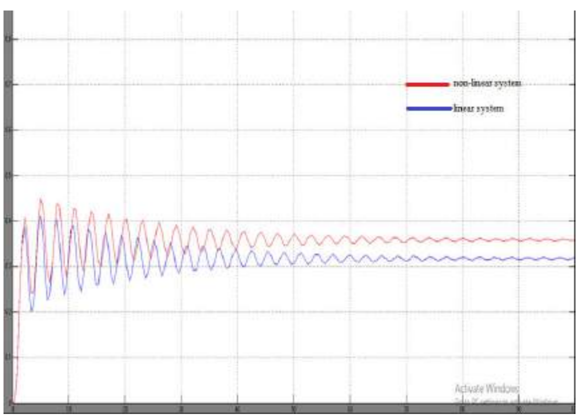
IV. SIMULATION RESULTS

A Twin Rotor MIMO System was modeled by considering 7 states and Zeigler Nichol's second method was used to design PID controllers and applied to the Simulink model of the system. All the simulations were carried out in Matlab Simulink. Step input was given as the references for both pitch and yaw rotors to study the response of PID controllers.

A. Response of open loop system

The open loop response of both the pitch and yaw rotors are shown in figure 4.1. we can see that both are showing the same characteristics. The following figure shows the open loop response. Here the green line represents then step input given to the two rotors of TRMS, red line shows the response of the non-linear system and the blue line represents the response of the linear system of TRMS. In the open loop, by providing a step input to the systems, they are showing stable but not tacking to the step value. Here we have given a step input of unity. Thus, we go for the closed loop system using PID controllers to reduce the tracking error to less than 1%.

Angle in degrees



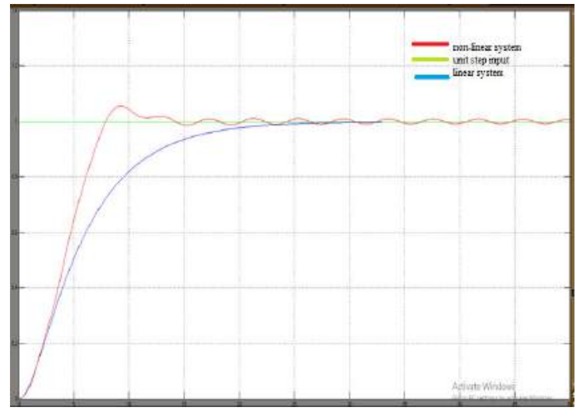
Time in sec

Fig. 4.1 Response of both systems towards a unit step input

B. Response of closed loop system

The response of the closed loop system given in figures 4.2 (a) and 4.2 (b) shows that both the pitch and yaw rotors are tracking the step input given to it. Here we can see that the response of pitch rotor is satisfactory as compared to that of the yaw rotor, this is because, the transfer function of pitch rotor is more complex as compared to that of the yaw rotor. However our closed loop system is found to be stable, and tracking errors are reduced to less than 1%.

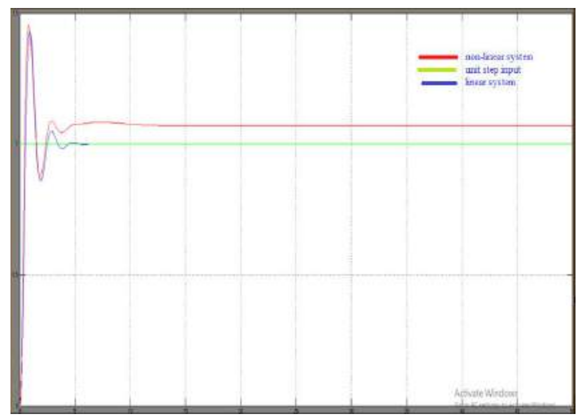
Angle in degrees



Time in sec

(a) Response of pitch rotor

Angle in degrees



Time in sec

(b) Response of yaw rotor

Fig. 4.2 Response of both systems towards a unit step input with PID controllers.

Thus our system is now ready to be implemented for control techniques such as Eigen Structure Assignment and Relative Gain Array algorithm techniques, which are selected as the future extension of the work.

V. CONCLUSIONS

A Twin Rotor MIMO system was modeled by considering 7 states. There are two inputs and two outputs for the TRMS, so it is a MIMO system having two degrees of freedom along the pitch and yaw angles of motion. PID controllers were designed for TRMS using Zeigler Nichol's tuning method. Thus, the tracking error of the closed loop system were reduced to less than 1% with the help of properly designed PID controllers for both linear as well as for the non-linear system of TRMS. The open loop and closed loop responses of both the systems were then compared and ensured the similarity in their responses. Now it is able to implement techniques of Eigen structure Assignment and RGA algorithm to the TRMS model that we have considered.

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